

# Area and Volume

## Chapter 10



### LESSONS

- 10-1** Area: Parallelograms
- 10-2** Area: Triangles and Trapezoids
- 10-3** Area: Circles
- 10-4** Space Figures
- 10-5** Surface Area: Prisms and Cylinders
- 10-6** Surface Area: Pyramids, Cones, and Spheres
- 10-7** Volume: Prisms and Cylinders
- 10-8** Problem Solving: Make a Model
- 10-9** Volume: Pyramids, Cones, and Spheres

### Key Vocabulary

- altitude (p. 523)
- altitude of a triangle (p. 527)
- area (p. 522)
- cone (p. 539)
- cubic unit (p. 557)
- cylinder (p. 539)
- lateral area (p. 546)
- net (p. 540)
- prism (p. 539)
- pyramid (p. 539)
- slant height (p. 552)
- space figure (p. 539)
- sphere (p. 539)
- surface area (p. 545)
- volume (p. 557)

## Where You're Going

In this chapter, you will learn how to

- Find the areas of figures.
- Find the surface areas of space figures.
- Find the volumes of space figures.
- Solve a problem by making a model.



**Real-World Snapshots** Applying what you learn, on pages 576–577 you will solve problems about playing fields.

# Area: Parallelograms

## What You'll Learn

OBJECTIVE 1

To find areas of rectangles

OBJECTIVE 2

To find areas of parallelograms

## ... And Why

To use formulas in finding areas of objects, such as the area of a banner

## Check Skills You'll Need

Use  $A = \ell w$  and find the third value.

- $A = 54 \text{ in.}^2, w = 6 \text{ in.}$
- $\ell = 35 \text{ m}, w = 7 \text{ m}$
- $A = 25 \text{ cm}^2, \ell = 2.5 \text{ cm}$
- $\ell = 7.2 \text{ ft}, w = 7.2 \text{ ft}$

For help, go to Lesson 3-4.

## New Vocabulary

- area
- altitude

## OBJECTIVE

1

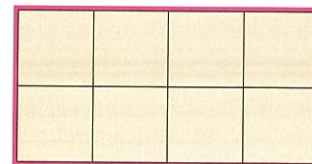
## Finding Areas of Rectangles

## Investigation

### Discovering an Area Formula

- Use a 3 in.-by-5 in. index card. Find the area of the card.
- Draw a line from one vertex to a point on another side to create a triangle. Cut along that line.
- Use the pieces to form a parallelogram that is not a rectangle.
- What is the area of your parallelogram? Explain.

The **area** of a figure is the number of square units it encloses. The rectangle outlined in red encloses 8 square units, each with area  $1 \text{ cm}^2$  (1 square centimeter). So, the area of the rectangle is  $8 \text{ cm}^2$ .



You can use the formula  $A = bh$  to find the area of a rectangle, where  $b$  is the length of one side and  $h$  is the length of the other. For the rectangle above,  $A = 4 \cdot 2 = 8$ . So, the area is  $8 \text{ cm}^2$ .

When you find area, the dimensions must be in the same unit.

## 1 EXAMPLE

## Real-World Problem Solving

**Parades** Find the area of the marching band's rectangular banner.

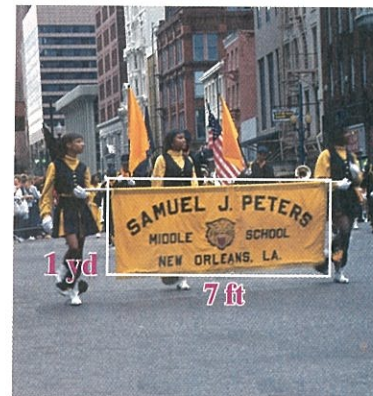
**Step 1** Change the units so that they are the same.

$$1 \text{ yd} = 3 \text{ ft} \quad \text{Change 1 yard to feet.}$$

**Step 2** Find the area.

$$\begin{aligned} A &= bh && \text{Use the formula for the area of a rectangle.} \\ &= (7)(3) && \text{Replace } b \text{ and } h \text{ with the dimensions 7 and 3.} \\ &= 21 && \text{Simplify.} \end{aligned}$$

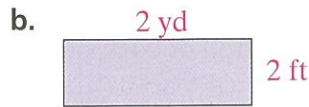
- The area of the banner is  $21 \text{ ft}^2$ .



**iTEXT** Interactive lesson includes instant self-check, tutorials, and activities.

**Check Understanding Example 1**

1. Find the area of each rectangle.



**OBJECTIVE**

**2**

**Finding Areas of Parallelograms**

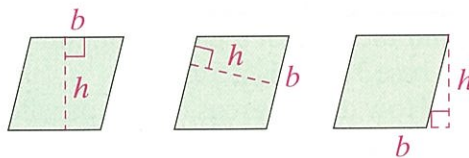
A rectangle is a special kind of parallelogram. The formula for the area of a parallelogram follows from the formula for the area of a rectangle. The height  $h$  of a parallelogram is the length of an *altitude*.

An **altitude** is a line segment perpendicular to the line containing a base of the figure and drawn from the side opposite that base.

**Key Concepts**

**Area of a Parallelogram**

The area of a parallelogram is the product of any base length  $b$  and the corresponding height  $h$ .



$$A = bh$$

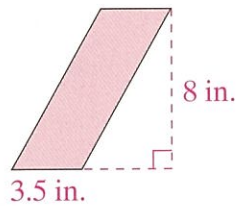
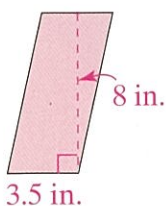
**Reading Math**

For any base in a figure, the *corresponding height* is the length of an altitude to that base.

**2 EXAMPLE**

**Finding Area of a Parallelogram**

Find the area of each parallelogram.

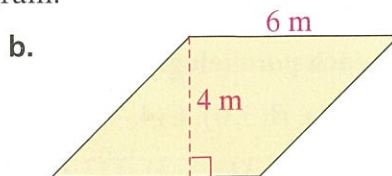
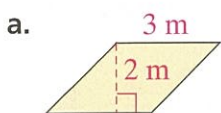


$$\begin{aligned} A &= bh && \text{area formula} \\ &= (3.5)(8) && \text{Substitute.} \\ &= 28 && \text{Simplify.} \end{aligned}$$

The area of each is  $28 \text{ in.}^2$ .

**Check Understanding Example 2**

2. Find the area of each parallelogram.



c. How do the areas of two parallelograms compare when the dimensions of one are twice the dimensions of the other?

# EXERCISES

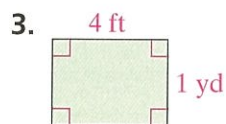
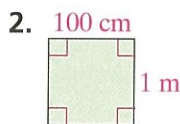
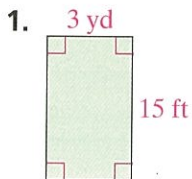
For more exercises, see *Extra Practice*.

## Practice and Problem Solving

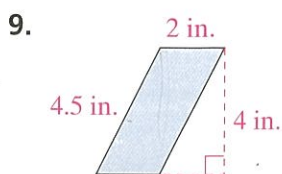
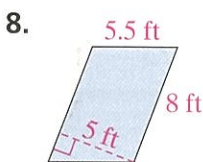
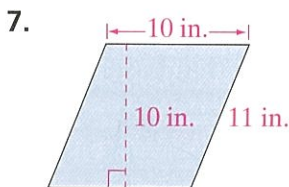
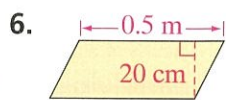
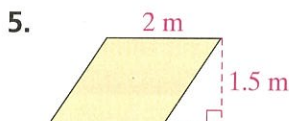
### A Practice by Example

Find the area of each parallelogram.

**Example 1**  
(page 522)



**Example 2**  
(page 523)

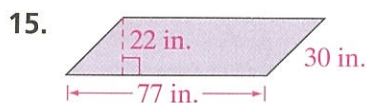
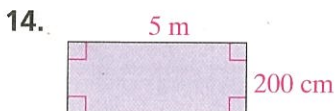
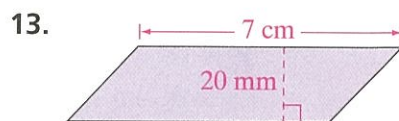
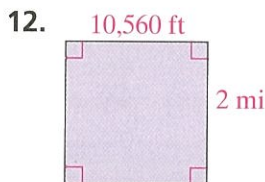


10. Find the area of a parallelogram with the given dimensions.
- base 3 cm, height 5 cm
  - base 9 cm, height 15 cm
  - How do the areas of two parallelograms compare when the dimensions of one are three times the dimensions of the other?

### B Apply Your Skills

11. **Sports** A football field is 300 ft long, from goal line to goal line, and 160 ft wide, from sideline to sideline. What is the area of a football field?

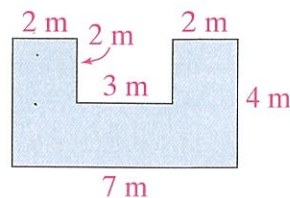
Find the area of each parallelogram.



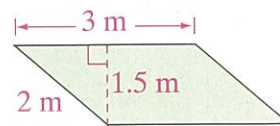
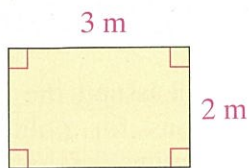
**Coordinate Geometry** The vertices of a parallelogram are given. Draw each parallelogram. Find its area.

16.  $A(0, 0), B(3, 0), C(4, 3), D(1, 3)$   
 17.  $W(-2, 0), X(-3, 3), Y(2, 0), Z(1, 3)$

18. Find the area of the figure at the right. Assume that all angles are right angles.



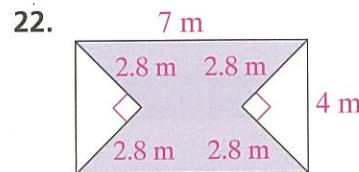
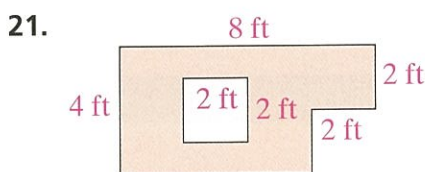
19. **Writing in Math** The two parallelograms below have the same perimeter. Are the areas the same? Explain.



**C Challenge**

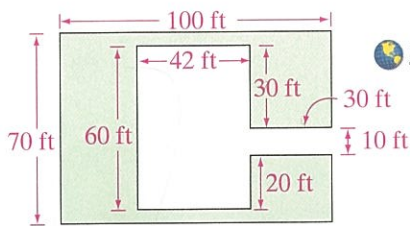
20. **Open-Ended** You want to make a  $400\text{-ft}^2$  vegetable garden. You plan to build a fence to keep the rabbits out. To spend the least amount of money, you want to use as little fencing as possible.
- Draw and list three possible dimensions for your garden.
  - Which of the three will need the least amount of fencing?

**Find the area of each shaded region. Assume that all angles that appear to be right angles are right angles.**



**Reasoning** In Exercises 23 and 24, find the ratio of the areas of the parallelograms (smaller to larger). Justify each answer.

23. The bases are the same length. The height of one parallelogram is twice the height of the other.
24. The height and the length of a base of one parallelogram are both twice those of the other parallelogram.



25. **a. Landscaping** Find the area of the yard at the left. Assume that all angles that appear to be right angles are right angles.
- b.** How many square yards of sod do you need to cover the yard?
- c.** One bag of fertilizer covers approximately  $2,000\text{ ft}^2$ . How many bags should you buy to cover the yard?



**Test Prep**

**Multiple Choice**

26. Square  $ABCD$  has perimeter  $x$  units. What is its area in terms of  $x$ ?
- A.  $\frac{x^2}{16}$  units<sup>2</sup>    B.  $4x^2$  units<sup>2</sup>    C.  $\frac{x^2}{4}$  units<sup>2</sup>    D.  $\frac{x}{16}$  units<sup>2</sup>
27. A parallelogram has base 18 cm and height 9 cm. What is its area?
- F.  $27\text{ cm}^2$     G.  $81\text{ cm}^2$     H.  $162\text{ cm}^2$     I.  $324\text{ cm}^2$
28. What is the side length of a square with area  $16\text{ cm}^2$ ?
- A.  $256\text{ cm}^2$     B.  $32\text{ cm}^2$     C. 8 cm    D. 4 cm
29. A rectangle has base  $r$  and height  $s$ . Which formula gives its area?
- F.  $A = r^2$     G.  $A = s^2$     H.  $A = rs$     I.  $A = 2r + 2s$

## Reading Comprehension

Read the passage below before doing Exercises 30 and 31.

### *Political Sign Regulations Set*

The City Council issued the following regulations for political signs in the upcoming election:

1. Only one sign may be posted on a single lot of land.
2. Sign areas may not exceed  $32 \text{ ft}^2$ .
3. Signs cannot be more than 6 ft in height.
4. All signs must be removed no later than 7 days after the election.



#### Take It to the NET

Online lesson quiz at  
[www.PHSchool.com](http://www.PHSchool.com)

Web Code: ada-1001

30. You wish to display a sign that is 6 ft high. What is the greatest width it can have?
31. A sign with area  $24 \text{ ft}^2$  is at least 1 ft high. Write an inequality that shows all possible values for its base length  $b$ .

## Mixed Review

### Lesson 9-10

The endpoints of a segment are given. Graph each segment and its image after a rotation of  $90^\circ$  about the origin.

32.  $A(5, 8), B(2, 4)$     33.  $C(0, 3), D(3, -5)$     34.  $E(-2, -3), F(-2, 4)$

### Lesson 9-2

35. Find the measures of two supplementary angles if the difference of their measures is  $56^\circ$ .

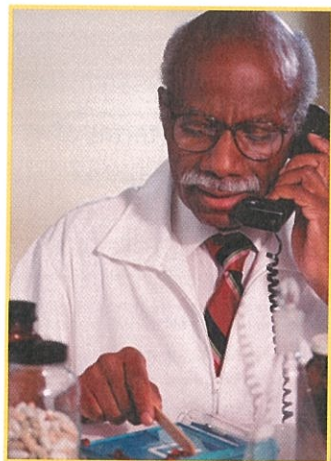
### Lesson 5-4

Simplify each expression.

36.  $\frac{2}{5} \cdot 2\frac{1}{2}$     37.  $\frac{2}{5} \div 2\frac{1}{2}$     38.  $3\frac{2}{3} \cdot \frac{3}{4}$     39.  $3\frac{2}{3} \div \frac{3}{4}$

## Math at Work

### Pharmacist



**Pharmacists** dispense medications. They also talk to their customers about the possible side effects of medications. To do this, they must thoroughly understand how prescription drugs are made.

Sometimes a doctor prescribes a special medication that a pharmacist must mix. The pharmacist must measure the ingredients in the exact proportions that the patient needs. In a situation such as this, mathematics is essential.



**Take It to the NET** For more information about pharmacists, go to [www.PHSchool.com](http://www.PHSchool.com).

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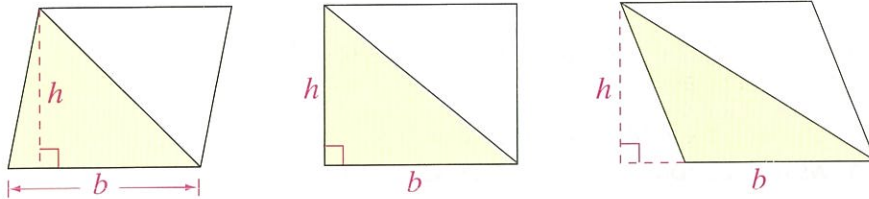
# Area: Triangles and Trapezoids

# 10-2

## OBJECTIVE

### 1 Finding Areas of Triangles

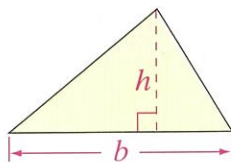
A diagonal divides a parallelogram into two congruent triangles.



You can see that the area of a triangle is half the area of a parallelogram. An **altitude of a triangle** is the perpendicular segment from a vertex of a triangle to the line containing the opposite side. The height is the length of the altitude.

### Key Concepts Area of a Triangle

The area of a triangle equals half the product of any base length  $b$  and the corresponding height  $h$ .



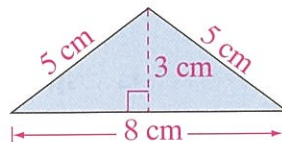
$$A = \frac{1}{2}bh$$

### 1 EXAMPLE Finding Area of a Triangle

Find the area of the triangle.

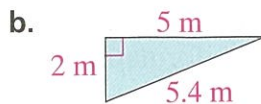
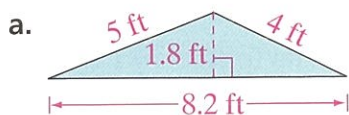
$$\begin{aligned} A &= \frac{1}{2}bh && \text{Use the formula for the area of a triangle.} \\ &= \frac{1}{2} \cdot 8 \cdot 3 && \text{Replace } b \text{ with 8 and } h \text{ with 3.} \\ &= 12 && \text{Simplify.} \end{aligned}$$

• The area is  $12 \text{ cm}^2$ .



### Check Understanding Example 1

1. Find the area of each triangle.



### What You'll Learn

**OBJECTIVE 1** To find areas of triangles

**OBJECTIVE 2** To find areas of trapezoids

### ... And Why

To find areas in real-world situations, such as construction

### Check Skills You'll Need

Find each product.

1.  $\frac{1}{2} \cdot 16$     2.  $\frac{1}{2} \cdot 14 \cdot 6$

3.  $\frac{1}{2} \cdot 5 \cdot 15$

4.  $\frac{1}{2} \cdot 2\frac{1}{2} \cdot 8$

For help, go to Lesson 5-4.

### New Vocabulary

- altitude of a triangle



## 2 EXAMPLE

### Real-World Problem Solving

**Construction** How much siding does a builder need to cover the side of the house shown at the left?

Area of triangle

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \cdot 16 \cdot 9 \\ &= 72 \end{aligned}$$

Area of rectangle

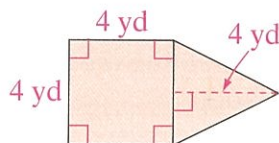
$$\begin{aligned} A &= bh \\ &= 16 \cdot 10 \\ &= 160 \end{aligned}$$

Add to find the total:  $72 + 160 = 232$ .

- The builder needs  $232 \text{ ft}^2$  of siding.

### Check Understanding Example 2

- Find the area of the shaded figure.



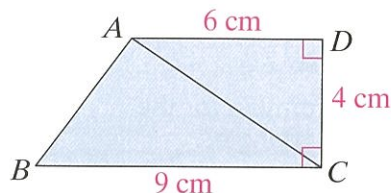
## OBJECTIVE

### 2

## Finding Areas of Trapezoids

A diagonal divides a trapezoid into two triangles. Notice that the triangles have the same height but different bases.

You can add the areas of the triangles to find the area of the trapezoid.



Area of  $\triangle ABC$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \cdot 9 \cdot 4 \\ &= 18 \end{aligned}$$

Area of  $\triangle ADC$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \cdot 6 \cdot 4 \\ &= 12 \end{aligned}$$

The areas of the two triangles are  $18 \text{ cm}^2$  and  $12 \text{ cm}^2$ . The area of the trapezoid is the sum of the areas of the two triangles,  $30 \text{ cm}^2$ .

You can use this information to find a formula for the area of a trapezoid.



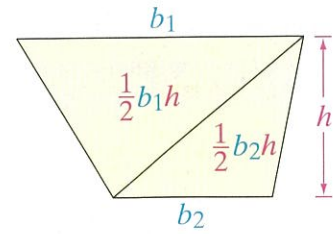
In a trapezoid, the parallel sides are its bases. For the figure at the right the bases are  $b_1$  and  $b_2$ . The height is  $h$ .

The area of the trapezoid is  $\frac{1}{2}b_1h + \frac{1}{2}b_2h$ .

By using the Distributive Property, you can see that

$$\frac{1}{2}b_1h + \frac{1}{2}b_2h \text{ is } \frac{1}{2}h(b_1 + b_2).$$

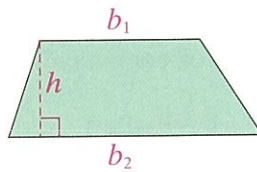
So, the area of the trapezoid is  $\frac{1}{2}h(b_1 + b_2)$ .



### Key Concepts Area of a Trapezoid

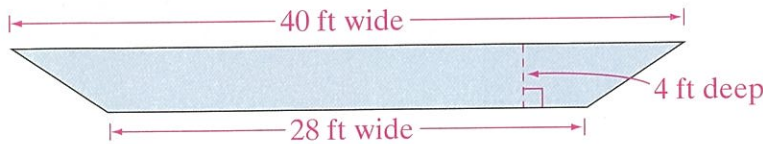
The area of a trapezoid is half the product of the height and the sum of the lengths of the bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$



### 3 EXAMPLE Real-World Problem Solving

**Erie Canal** The Erie Canal opened in 1825 and was hailed as an engineering marvel. Below is a cross section of the Erie Canal. Find the area of the trapezoidal cross section.



$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Use the formula for the area of a trapezoid.}$$

$$A = \frac{1}{2} \cdot 4(28 + 40) \quad \text{Replace } h \text{ with } 4, b_1 \text{ with } 28, \text{ and } b_2 \text{ with } 40.$$

$$= \frac{1}{2} \cdot 4(68) \quad \text{Simplify.}$$

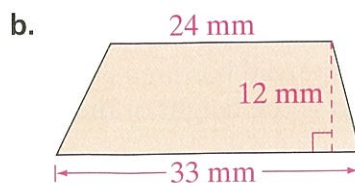
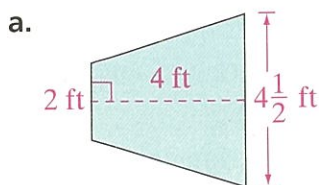
$$= 2 \cdot 68$$

$$= 136$$

The area of the cross section is  $136 \text{ ft}^2$ .

### Check Understanding Example 3

3. Find the area of each trapezoid.



### Real-World Connection

The Erie Canal is 363 miles in length.

# EXERCISES

For more exercises, see *Extra Practice*.

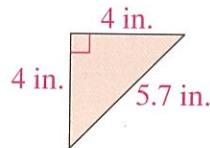
## Practice and Problem Solving

### A Practice by Example

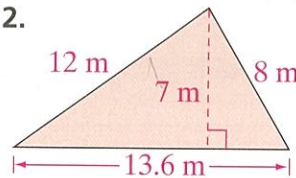
Find the area of each triangle.

**Example 1**  
(page 527)

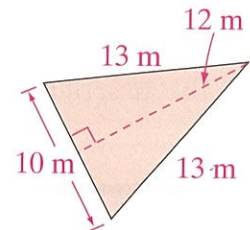
1.



2.



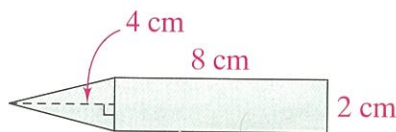
3.



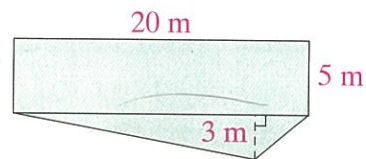
**Example 2**  
(page 528)

Find the area of each shaded figure.

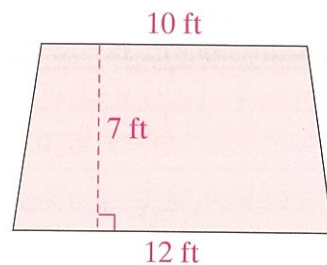
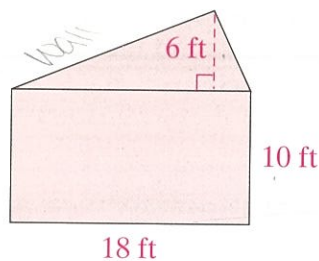
4.



5.



- 6. Interior Design** A designer wants to cover the wall shown at the left below with wallpaper. How much wallpaper does she need?

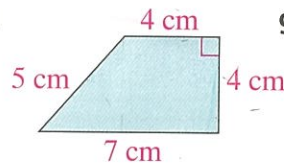


**Example 3**  
(page 529)

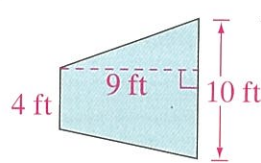
- 7. Shipping** One end of an air cargo container has the shape shown at the right above. Find the area of the trapezoid.

Find the area of each trapezoid.

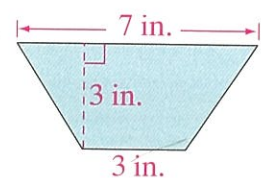
8.



9.



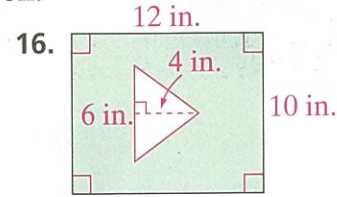
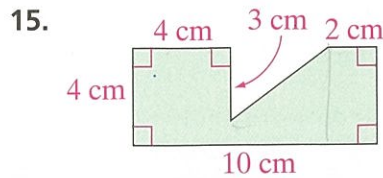
10.



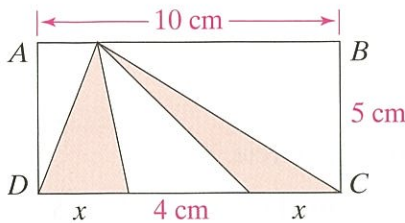
### B Apply Your Skills

- Reasoning** A triangle and a parallelogram both have areas of  $20 \text{ cm}^2$  and bases of 5 cm. How do their heights compare?
- Writing in Math** Are  $(\frac{1}{2} \cdot 3) \cdot 8$  and  $3 \cdot (\frac{1}{2} \cdot 8)$  equal? Explain how this can help in finding the area of a triangle.
- A trapezoid has area  $50 \text{ in.}^2$ . The two bases are 5 in. and 15 in. What is the height of the trapezoid?
- Open-Ended** Sketch and label two different triangles so that both have areas of  $180 \text{ in.}^2$ .

Find the area of each shaded region.



**C Challenge**



**Reasoning** In Exercises 17 and 18, find the ratios of the areas of the triangles (smaller to larger). Justify each answer.

17. The height and the lengths of the bases of one trapezoid are all twice those of the other trapezoid.  
 18. The trapezoids have the same height. One trapezoid has base lengths  $b$  and  $2b$ . The other has base lengths  $b$  and  $4b$ .  
 19.  $ABCD$  (left) is a rectangle. What is the area of the shaded region?



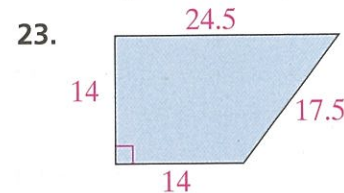
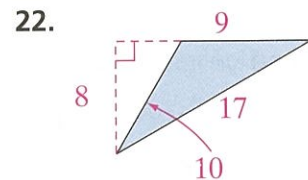
**Test Prep**

**Multiple Choice**

20. What are possible dimensions for a triangle with area  $32 \text{ cm}^2$ ?  
 A.  $b = 4 \text{ cm}$     B.  $b = 2 \text{ cm}$     C.  $b = 8 \text{ cm}$     D.  $b = 6 \text{ cm}$   
 $h = 8 \text{ cm}$      $h = 16 \text{ cm}$      $h = 8 \text{ cm}$      $h = 8 \text{ cm}$
21. A trapezoid has an area of  $56 \text{ in}^2$ . One base is 6 in. and the height is 8 in. What is the length of the other base?  
 F. 6 in.    G. 8 in.    H. 10 in.    I. 14 in.

**Short Response**

What is the number of square units in each figure? Show your work.

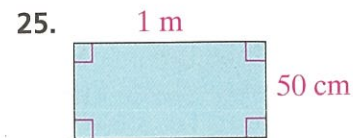
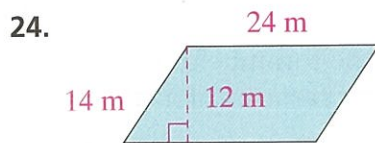


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 Online lesson quiz at  
[www.PHSchool.com](http://www.PHSchool.com)  
 Web Code: ada-1002

**Mixed Review**

**Lesson 10-1**

Find the area of each parallelogram.



**Lessons 7-1 and 7-3**

Solve each equation.

26.  $16 + 2y = 9$     27.  $5.5a + 2 = 10.5$     28.  $\frac{1}{2}b - 10 = 24$

**Lesson 6-9**

29. Suppose you bought two books for \$15. The original prices of the two books were the same, but you were able to buy one for half price. What was the full price of each book?

# Area: Circles

**OBJECTIVE**
**1**
**Finding Areas of Circles**
**What You'll Learn**
**OBJECTIVE**
**1**

To find areas of circles

**OBJECTIVE**
**2**

To find areas of irregular figures that include parts of circles

**... And Why**

To use area formulas in real-world situations, such as finding the amount of grass needed to cover a circular region

**Check Skills You'll Need**

Simplify each expression.

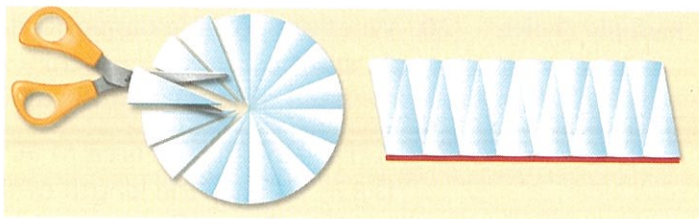
- $3.14 \cdot 4^2$
- $3.14 \cdot 5^2$
- $3.14 \cdot 9^2$
- $3.14 \cdot 0.5^2$

 For help, go to Lesson 4-2.

## Investigation

### Finding the Formula for Area of a Circle

- Use a compass to draw a circle. Cut out the circle.
- Fold the circle in half, and then in half again. Fold it in half a third and fourth time.
- Cut out the 16 wedges that you have formed with the folds.
- Arrange the wedges in a row as shown below.



- Notice that the new shape resembles a parallelogram. How does the base of the parallelogram (the side shown in red) relate to the circumference of the circle? How does the height of the parallelogram relate to the radius of the circle?
- Use the formula for the area of a parallelogram to estimate the area of your circle.

The diagram above shows the relationship between the area of a circle and a figure that is like a parallelogram. The height  $h$  of the parallelogram is about the same as the radius  $r$  of the circle. The base  $b$  is about half the circumference  $C$  of the circle.


You can use the formula for the area of a parallelogram to suggest the formula for the area of a circle.

$A = bh$       Use the formula for the area of a parallelogram.

$A = \left(\frac{1}{2}C\right)(r)$       Substitute  $\frac{1}{2}C$  for  $b$  and  $r$  for  $h$ .

$A = \frac{1}{2}(2\pi r) \cdot r$       Substitute  $2\pi r$  for  $C$ .

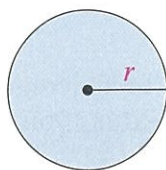
$A = \pi r^2$       Simplify.

 **iTEXT** Interactive lesson includes instant self-check, tutorials, and activities.

## Key Concepts Area of a Circle

The area of a circle equals the product of  $\pi$  and the square of the radius  $r$ .

$$A = \pi r^2$$



You can express the *exact area* of a circle by using  $\pi$ .

### 1 EXAMPLE Finding Area of a Circle

Find the exact area of a circle with diameter 12 cm.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(6)^2 \quad r = \frac{1}{2}d; r = 6 \\ &= 36\pi \quad \text{Simplify.} \end{aligned}$$

- The area is  $36\pi \text{ cm}^2$ .

#### ✓ Check Understanding Example 1

1. Find the exact area of a circle with radius 50 in.

For real-world situations, you usually want an approximate value for the area of a circle. If you are finding an approximate area, use 3.14 or the value provided by a calculator for  $\pi$ .

### 2 EXAMPLE Real-World Problem Solving

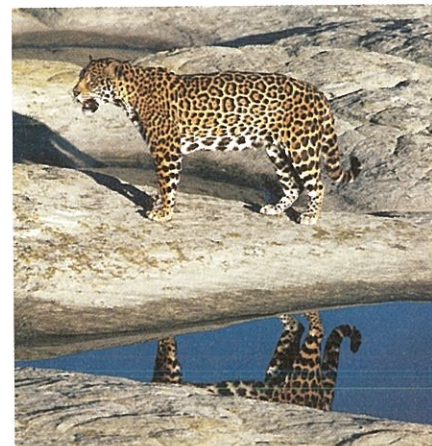
**Biology** The size of a jaguar's territory depends on how much food is available. In a situation where there is plenty of food, the circular territory of the jaguar may be as small as 3 mi in diameter. Find the area of such a region to the nearest square mile.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(1.5)^2 \quad r = \frac{1}{2}d; r = 1.5 \\ &= 2.25\pi \quad \text{exact area} \\ &\approx (2.25)(3.14) \quad \text{Use 3.14 for } \pi. \\ &= 7.065 \quad \text{approximate area} \end{aligned}$$

- The area of the region is about  $7 \text{ mi}^2$ .

#### ✓ Check Understanding Example 2

2. Find the approximate area of a circle with radius 6 mi.



#### Real-World Connection

During a drought a jaguar may need to search for food in a territory as large as 6 mi in diameter.

OBJECTIVE

2

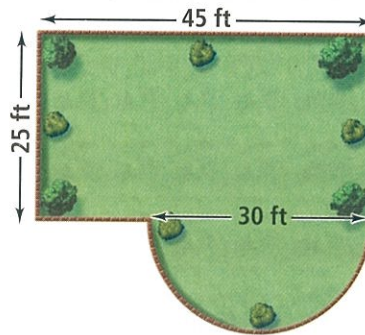
Finding Areas of Irregular Figures

To find the area of an irregular figure, you can sometimes separate it into figures with areas you know how to find.

3 EXAMPLE

Real-World Problem Solving

**Landscaping** A pound of grass seed covers approximately  $675 \text{ ft}^2$ . Find the area of the lawn below. Then find the amount of grass seed you need to buy to cover the lawn. Grass seed comes in 3-lb bags.



Area of region that is one half of a circle

$$\text{area of circle} = \pi r^2$$

$$\text{area of half circle} = \frac{1}{2}\pi r^2$$

$$A \approx \frac{1}{2}(3.14)(15)^2$$

Replace  $\pi$  with 3.14 and  $r$  with 15.

$$A = 353.25$$

Area of region that is a rectangle

$$\text{area of rectangle} = bh$$

$$A = 45 \cdot 25$$

Replace  $b$  with 45 and  $h$  with 25.

$$A = 1,125$$

The area of the lawn is about  $353 \text{ ft}^2 + 1,125 \text{ ft}^2 = 1,478 \text{ ft}^2$ .

$$1,478 \div 675 \approx 2.19$$

Divide to find the amount of seed.

- You need to buy one 3-lb bag of grass seed.



Need Help?

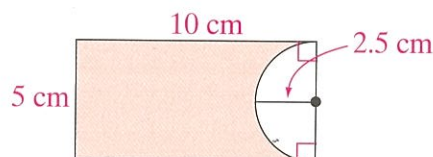
To find how much grass seed, use dimensional analysis:

$$\text{ft}^2 \div \frac{\text{ft}^2}{\text{lb}} =$$

$$\text{ft}^2 \cdot \frac{\text{lb}}{\text{ft}^2} = \text{lb}$$

Check Understanding Example 3

- Find the area of the shaded region to the nearest tenth.



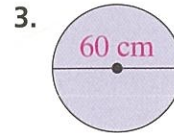
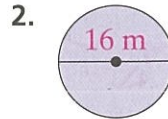
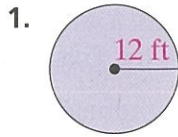
# EXERCISES

For more exercises, see *Extra Practice*.

## Practice and Problem Solving

**A Practice by Example**  
Examples 1 and 2  
(page 533)

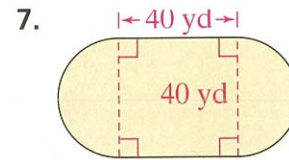
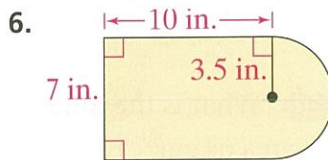
Find the area of each circle. Give the exact area, and an approximate area to the nearest square unit.



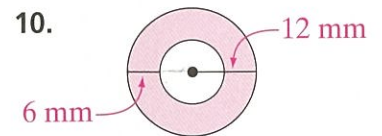
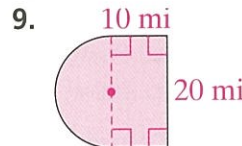
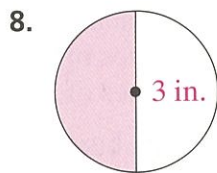
4. **Recreation** A standard dartboard has a diameter of 18 in. What is its area to the nearest square inch?
5. **Culinary Arts** A culinary student decorates an 8-in.-diameter round cake. What is the approximate area of the top of the cake?

**Example 3**  
(page 534)

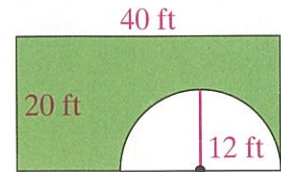
Find the area of each figure to the nearest square unit.



Find the area of each shaded region to the nearest square unit.



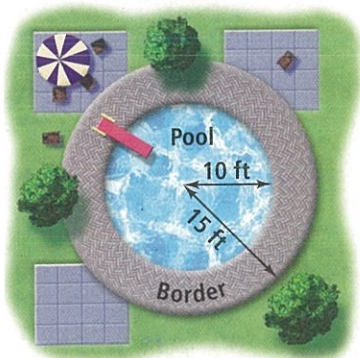
11. **Lawn Care** A groundskeeper wants to use sod to cover the lawn shown in green in the diagram. Each piece of sod covers 3 ft<sup>2</sup>. About how many pieces of sod are needed?



**B Apply Your Skills**

**Number Sense** Match each object with its most likely area.

- |   |                            |
|---|----------------------------|
| 12. dinner plate                            | A. 0.8 in. <sup>2</sup>    |
| 13. quarter                                 | B. 110 in. <sup>2</sup>    |
| 14. 12-in. circular pizza                   | C. 7 in. <sup>2</sup>      |
| 15. jar lid                                 | D. 16,000 in. <sup>2</sup> |
| 16. the center circle on a basketball court | E. 80 in. <sup>2</sup>     |



17. **Pool Design** Covering for the pool border is sold by the square yard (whole-number amounts only). It costs \$20/yd<sup>2</sup>.
- What is the area of the border in square feet?
  - About how much will the covering cost?

**Find the area of each circle. Give the exact area, and an approximate area to the nearest tenth.**

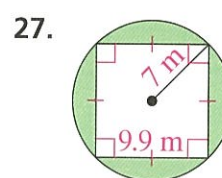
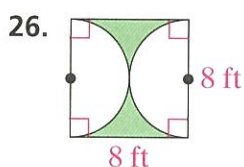
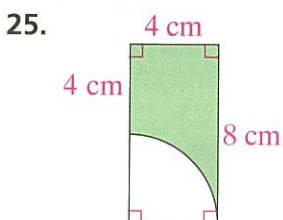
18.  $r = 12$  mi    19.  $r = 0.3$  m    20.  $d = 1.5$  in.    21.  $d = 8.4$  mm

22. **Open-Ended** Describe a real-life situation, not used in this lesson, where you might use the formula for the area of a circle.

23. Which has a greater area, four circles, each with radius 1 m, or one circle with radius 4 m? Explain.

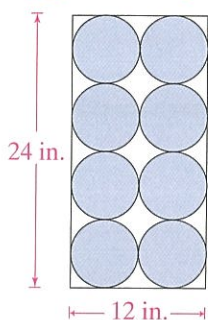
24. How many circles with radius 2 cm will have the same total area as a circle with radius 4 cm?

**Find the area of each shaded region to the nearest square unit.**



28. **Writing in Math** What is the area of the largest circle that will fit in a square with area  $64 \text{ cm}^2$ ? Explain.

**C Challenge**



29. **Can Lids** A manufacturer cuts lids for eight cans from one rectangular sheet of aluminum as shown at the left.

- What is the radius of each lid?
- How many square inches of aluminum do the lids require?
- How many square inches of aluminum are wasted?

30. **Consumer Issues** You can buy a 10-in. diameter pizza for \$6.50, a 12-in. pizza for \$8.50, or a 14-in. pizza for \$10.50.

- What is the area of each pizza to the nearest square inch?
- What is the price per square inch of each pizza?
- Reasoning** Is the largest pizza the best buy? Explain.



**Test Prep**

**Multiple Choice**

For Exercises 31–33, what is the exact area of each circle having radius  $r$  or diameter  $d$ ?

31.  $r = 11$  mi  
 A.  $5.5\pi \text{ mi}^2$     B.  $11\pi \text{ mi}^2$     C.  $22\pi \text{ mi}^2$     D.  $121\pi \text{ mi}^2$
32.  $r = \frac{1}{2}$  m  
 F.  $\frac{1}{4}\pi \text{ m}^2$     G.  $\frac{1}{2}\pi \text{ m}^2$     H.  $\pi \text{ m}^2$     I.  $2\pi \text{ m}^2$
33.  $d = 1.2$  in.  
 A.  $2.4\pi \text{ in.}^2$     B.  $1.44\pi \text{ in.}^2$     C.  $0.36\pi \text{ in.}^2$     D.  $0.18\pi \text{ in.}^2$



**Take It to the NET**

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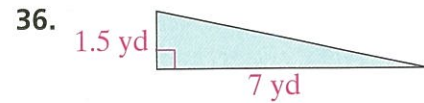
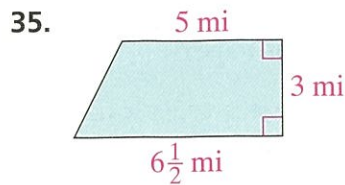
**Short Response**

34. Which has a greater area, (a) a circle with radius 2 m or a square with side length 2 m? (b) Explain your answer.

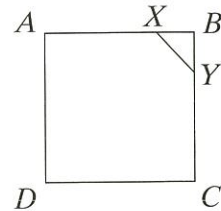


## Mixed Review

**Lesson 10-2** Find the area of each figure.



**Lesson 9-4** 37. Square  $ABCD$  has side length 8 in.  $\triangle BXY$  is isosceles. Its congruent sides have length 2 in. How many triangles congruent to  $\triangle BXY$  can you cut from square  $ABCD$ ?



**Lesson 5-3** Find each sum or difference.

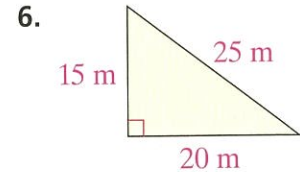
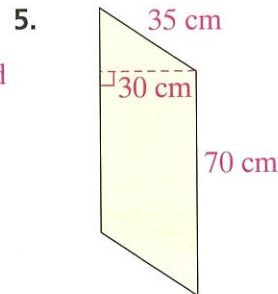
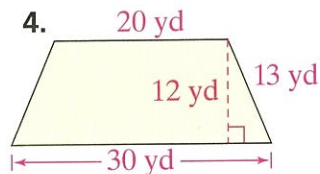
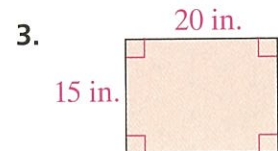
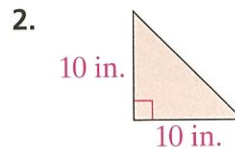
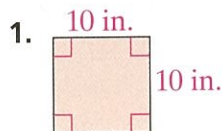
38.  $4\frac{3}{5} + 5\frac{2}{3}$     39.  $5\frac{2}{3} - 4\frac{3}{5}$     40.  $\frac{7}{8} + \frac{5}{6}$     41.  $\frac{7}{8} - \frac{5}{6}$

### Checkpoint Quiz 1

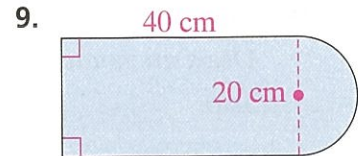
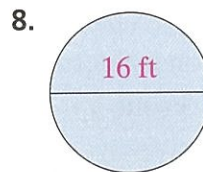
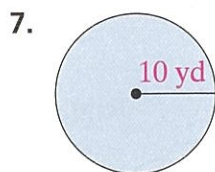
### Lessons 10-1 through 10-3

**TEXT** Instant self-check quiz online and on CD-ROM

Find the area of each figure.

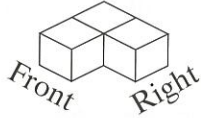


Find the area of each figure. Give the exact area, and an approximate area to the nearest square unit.



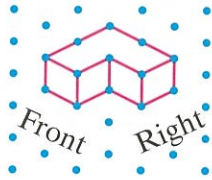
10. A trapezoid has an area of  $45 \text{ cm}^2$ . The two bases are 6 cm and 12 cm. What is the height of the trapezoid?

A solid is a three-dimensional figure.



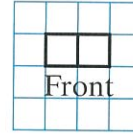
You can draw a solid *in perspective* (p. 542) to show that it is three-dimensional.

Isometric Dot Paper



You can use isometric dot paper to draw a three-dimensional view.

Rectangular Graph Paper

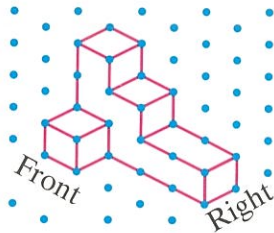


You can use rectangular graph paper to draw top, front, and side views.

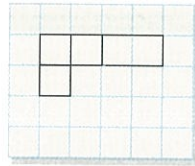
## EXAMPLE

Draw the top, front, and right-side views of the solid.

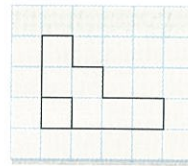
Isometric



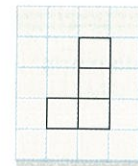
Top



Front



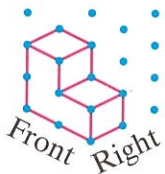
Right Side



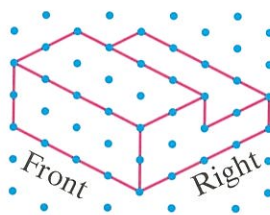
## EXERCISES

Draw the top, front, and right-side views of each solid.

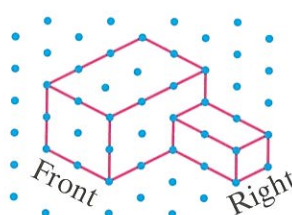
1.



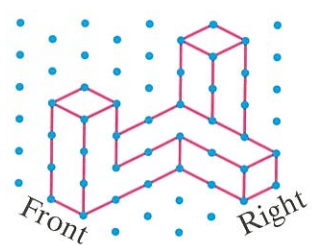
2.



3.



4.



5. The top, front, and right-side views are given. Draw an isometric view on isometric dot paper.

Top



Front



Right Side



# Space Figures

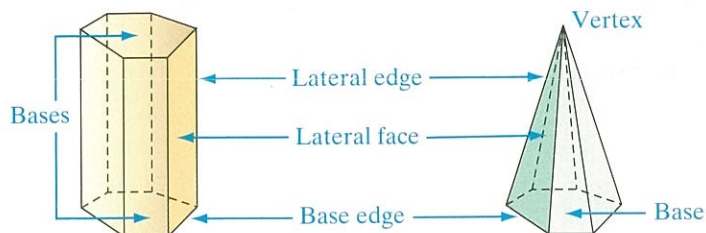
# 10-4

## OBJECTIVE

# 1

## Naming Space Figures

The figures below are common three-dimensional figures, also called **space figures** or solids. The space figures you will study in this book are prisms, pyramids, cylinders, cones, and spheres.

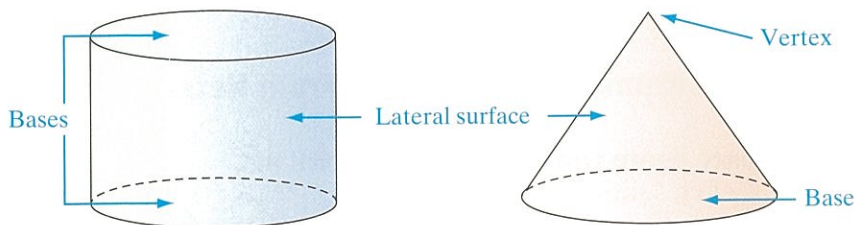


**Prism**

**Pyramid**

A **prism** has two parallel bases that are congruent polygons, and lateral faces that are parallelograms.

A **pyramid** has a base that is a polygon. The lateral faces are triangles.



**Cylinder**

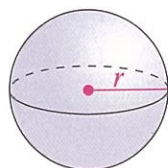
**Cone**

A **cylinder** has two parallel bases that are congruent circles.

A **cone** has one circular base and one vertex.

## Sphere

A **sphere** is the set of all points in space that are a given distance from a given point called the center.



## What You'll Learn

OBJECTIVE 1

To identify common space figures

OBJECTIVE 2





To identify space figures from nets

## ... And Why

To identify the space figures often used in constructing buildings

## Check Skills You'll Need

Judging by appearance, classify each polygon.

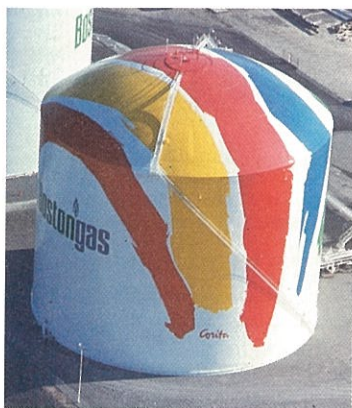
1. 
2. 
3. 
4. 

For help, go to Lesson 9-3.

## New Vocabulary

- space figures
- prism
- pyramid
- cylinder
- cone
- sphere
- net

**TEXT** Interactive lesson includes instant self-check, tutorials, and activities.



### Real-World Connection

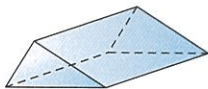
This space figure displays the 73,000-ft<sup>2</sup> "Rainbow" by Corita Kent.

You can use the shape of a base to help you name a space figure.

## 1 EXAMPLE Naming Space Figures

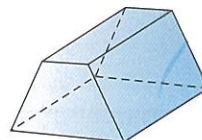
For each figure, describe the bases and name the figure.

a.



The bases are triangles.  
The figure is a triangular prism.

b.

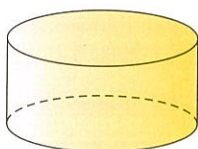


The bases of the prism are trapezoids. The figure is a trapezoidal prism.

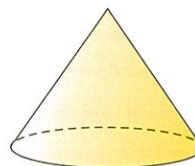
### Check Understanding Example 1

1. Name each figure.

a.



b.



## OBJECTIVE

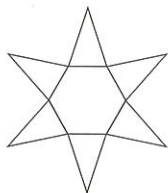
## 2 Identifying Space Figures From Nets

A **net** is a pattern you can form into a space figure.

## 2 EXAMPLE Naming Space Figures From Nets

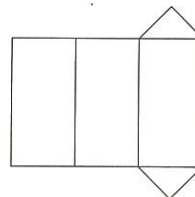
Name the space figure you can form from each net.

a.



With a hexagonal base and triangular sides, you can form a hexagonal pyramid.

b.

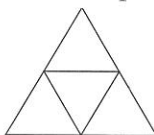


With two triangular bases and rectangular sides, you can form a triangular prism.

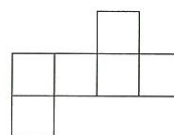
### Check Understanding Example 2

2. Name the space figure you can form from each net.

a.



b.



### Need Help?

Recall that a *cube* is a rectangular prism with six congruent square faces.

# EXERCISES

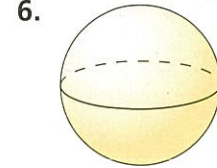
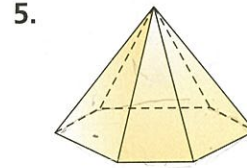
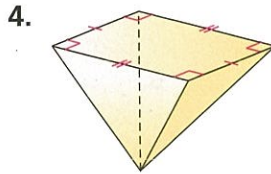
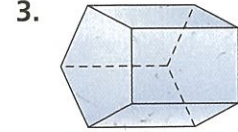
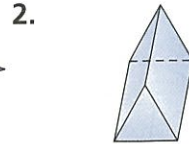
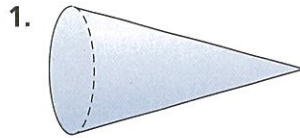
For more exercises, see *Extra Practice*.

## Practice and Problem Solving

### A Practice by Example

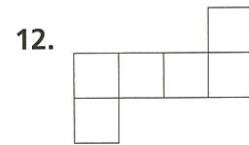
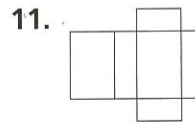
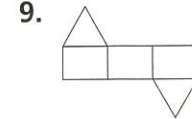
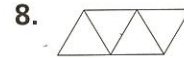
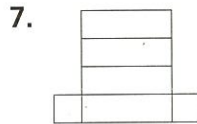
**Example 1**  
(page 540)

For each figure, describe the base(s), if any, and name the figure.



**Example 2**  
(page 540)

Name the space figure you can form from each net.



### B Apply Your Skills

#### Reading Math

*Lateral* means "on the side." The lateral faces of a prism or pyramid are the surfaces that connect with a base. See page 539.

For Exercises 13–15, write the most precise name for each space figure that has the given properties.

13. four lateral faces that are triangles
14. three lateral faces that are rectangles
15. a lateral surface and one circular base
16. What type of space figure does each object suggest?
  - a. a shoe box
  - b. a teepee
  - c. a basketball

**Open Ended** Draw a net for each space figure.

17. pentagonal pyramid
18. an object in your classroom

19. **Error Analysis** A student explains that since each figure below has six square faces, each can be folded to make a cube. Explain the error the student might have made.

Figure A

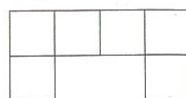


Figure B

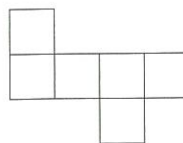


Figure C

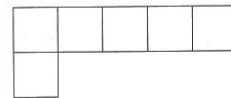
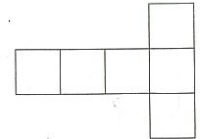


Figure D



20. A net is made of 4 congruent rectangles and 2 congruent squares whose sides are the same length as the shorter sides of the rectangles. Name a space figure you can form from this net.

Match each container with the correct net.

21.



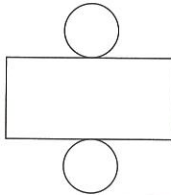
22.



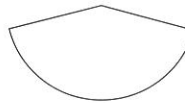
23.



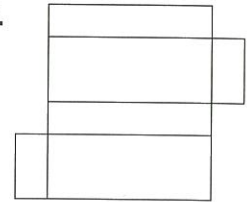
A.



B.



C.

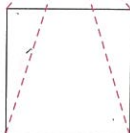


24. **Writing in Math** Suppose you see a net for a rectangular prism and a net for a rectangular pyramid. Explain how you can match each net with its name.
25. Draw a net to represent a rectangular box that is 10 cm long, 8 cm wide, and 4 cm high. Label dimensions on the net.
26. A cube is easy to draw in *one-point perspective*. Draw a cube in one-point perspective by following the steps below.

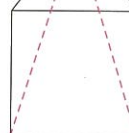
**C Challenge**



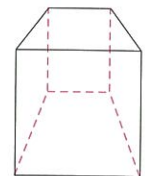
Step 1



Step 2



Step 3



Step 4

**Step 1** Begin by drawing a square for the front. Draw a *horizon line* parallel to one horizontal edge of your square. Select a *vanishing point* on the horizon line.

**Step 2** Draw lines, called *vanishing lines*, from the vertices of the square to the vanishing point.

**Step 3** Draw a line segment parallel to the horizon line. Use this segment to determine the top and back edges.

**Step 4** Draw dashed lines for the hidden back vertical and horizontal edges. Erase the horizon line and unnecessary parts of the vanishing lines.

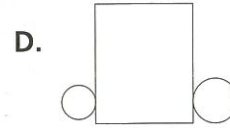
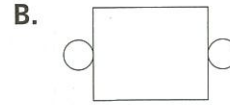
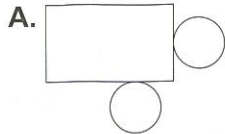
27. Using steps like those suggested in Exercise 26, make a one-point perspective drawing of each figure.
- triangular prism
  - cylinder



# Test Prep

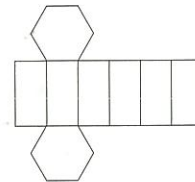
## Multiple Choice

28. Which of the following is a net for a cylinder?



29. Which space figure can you form from the net?

- F. hexagonal pyramid
- G. triangular pyramid
- H. triangular prism
- I. hexagonal prism



30. Which space figure has three lateral faces that are triangles?

- A. triangular pyramid
- B. pentagonal prism
- C. square prism
- D. triangular prism

31. Which space figure has 6 faces that are all regular quadrilaterals?

- F. triangular pyramid
- G. pentagonal prism
- H. square prism
- I. triangular prism

32. A rectangular prism has how many faces?

- A. 2
- B. 4
- C. 6
- D. 8



### Take It to the NET

Online lesson quiz at [www.PHSchool.com](http://www.PHSchool.com)  
Web Code: ada-1004

## Short Response

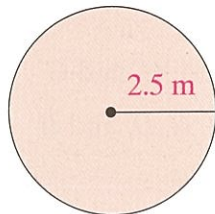
33. a. Draw a net for a cylinder that has diameter 4 in. and height 6 in.  
b. Label your drawing.

## Mixed Review

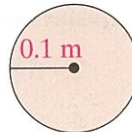
### Lesson 10-3

For each circle, find the exact area and an approximate area.

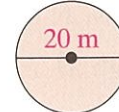
34.



35.



36.



### Lessons 9-4

37. **Mowing Lawns** A rectangular yard is 20 ft by 40 ft. Your lawn mower will mow a 2-ft-wide path. What is the least number of turns you must make to mow the lawn?

### Lesson 8-3

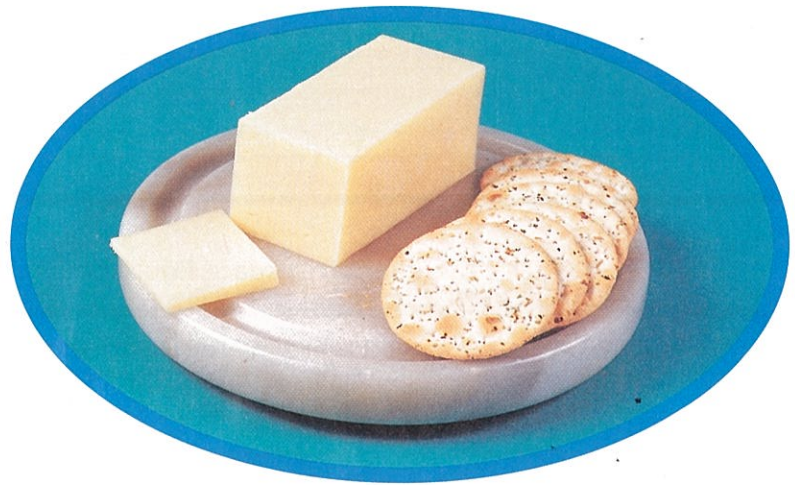
Write each equation in slope-intercept form.

38.  $3x - y = 6$

39.  $2x - 2y = 10$

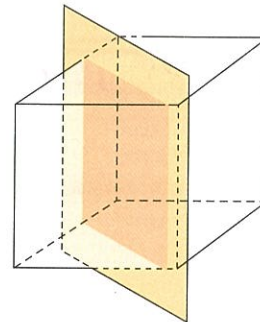
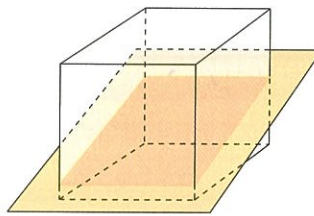
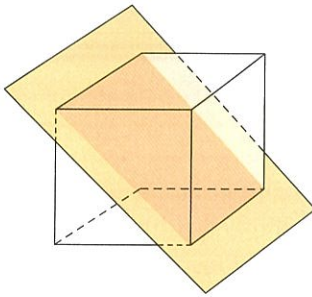
40.  $-8y - 16 = 24x$

The intersection of a plane and a space figure is a *cross section* of the space figure. This cross section of a block of cheese is a rectangle.



**EXAMPLE**

Sketch a plane intersecting a cube in three different ways to show a rectangular cross section.



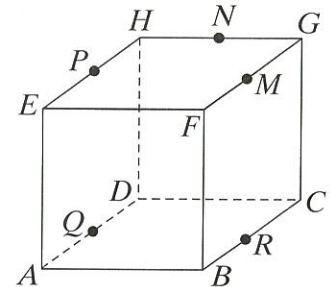
**EXERCISES**

Use the name of a polygon to describe each cross section of the cube. Points  $M$ ,  $N$ ,  $P$ ,  $Q$ , and  $R$  are midpoints of edges.

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| 1. through $M$ , $P$ , $Q$ , and $R$ | 2. through $E$ , $A$ , $C$ , and $G$ |
| 3. through $B$ , $E$ , and $G$       | 4. through $M$ , $N$ , $D$ , and $B$ |

Sketch a cube to show each cross section.

- |                       |                          |
|-----------------------|--------------------------|
| 5. a scalene triangle | 6. a trapezoid           |
| 7. a square           | 8. an isosceles triangle |
9. Describe the possible cross sections of a sphere.
10. Sketch and describe two possible cross sections of a cylinder.





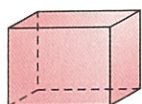
# Surface Area: Prisms and Cylinders

10-5

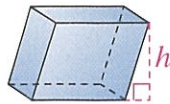
OBJECTIVE

## 1 Finding Surface Areas of Prisms

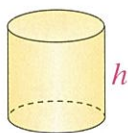
Prisms and cylinders can be *right* or *oblique*.



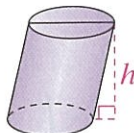
Right prism



Oblique prism



Right cylinder



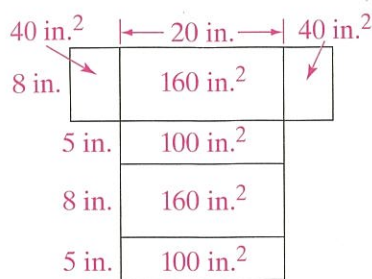
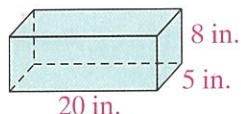
Oblique cylinder

In this text, you may assume that prisms and cylinders are right unless otherwise stated.

**Surface area (S.A.)** is the sum of the areas of the base(s) and the lateral faces of a space figure. One way to find the surface area of a space figure is to find the area of its net. You measure surface area in square units.

### 1 EXAMPLE Finding Surface Area Using a Net

Find the surface area of the rectangular prism using a net.



Draw and label a net.

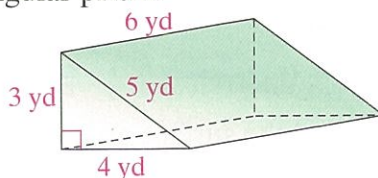
Find the area of each rectangle in the net.

$$40 + 40 + 160 + 100 + 160 + 100 = 600 \quad \text{Add the areas.}$$

The surface area is  $600 \text{ in.}^2$ .

### Check Understanding Example 1

- Find the surface area of the triangular prism.
- A similar triangular prism has dimensions twice those shown here. Find its surface area.
- How do surface areas of similar prisms compare when dimensions are doubled?



### What You'll Learn

OBJECTIVE 1 To find surface areas of prisms

OBJECTIVE 2 To find surface areas of cylinders

### ... And Why

To find the amount of material needed in packaging

### Check Skills You'll Need

Find the circumference of each circle with the given radius or diameter.

- $r = 5 \text{ in.}$
- $r = 4.2 \text{ cm}$
- $d = 8 \text{ ft}$
- $d = 6.8 \text{ in.}$

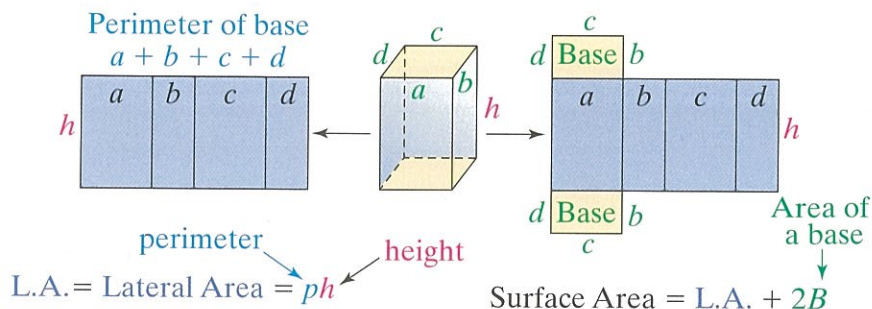
For help, go to Lesson 9-6.

### New Vocabulary

- surface area
- lateral area

**TEXT** Interactive lesson includes instant self-check, tutorials, and activities.

Another way to find the surface area of a prism is to use the *lateral area* and the base areas. **Lateral area (L.A.)** of a prism is the sum of the areas of the lateral faces.



To find surface area, it is a good idea to find lateral area first.

### Key Concepts

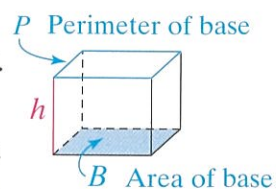
### Surface Area of a Prism

The lateral area of a prism is the product of the perimeter of the base and the height.

$$L.A. = ph$$

The surface area of a prism is the sum of the lateral area and the areas of the two bases.

$$S.A. = L.A. + 2B$$



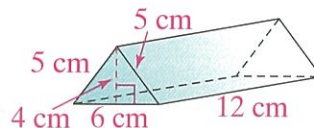
#### Need Help?

$B$  represents the area of a base. For a list of area formulas, see the table on page 780.

## 2 EXAMPLE

### Finding Surface Area Using Formulas

Find the surface area of the triangular prism at the right.



**Step 1** Find the lateral area.

$$\begin{aligned} L.A. &= ph \\ &= (5 + 5 + 6)12 \\ &= 192 \end{aligned}$$

Use the formula for lateral area.

$$p = 5 + 5 + 6 \text{ and } h = 12.$$

**Step 2** Find the surface area.

$$\begin{aligned} S.A. &= L.A. + 2B \\ &= 192 + 2\left(\frac{1}{2} \cdot 6 \cdot 4\right) \\ &= 192 + 24 \\ &= 216 \end{aligned}$$

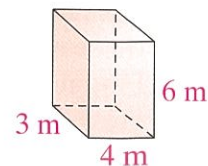
Use the formula for surface area.

$$L.A. = 192 \text{ and } B = \frac{1}{2} \cdot 6 \cdot 4.$$

- The surface area of the triangular prism is  $216 \text{ cm}^2$ .

### ✓ Check Understanding Example 2

2. Find the surface area of the prism at the right.

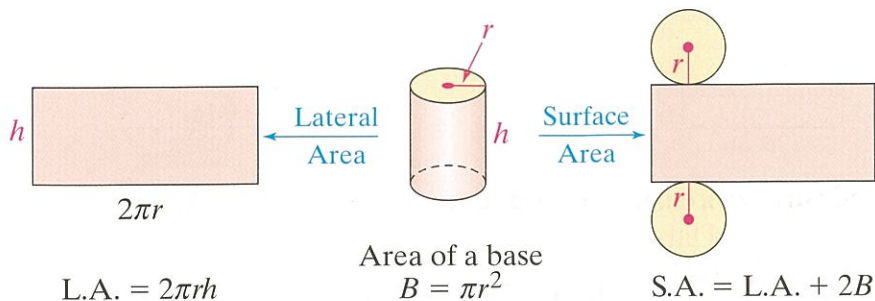


OBJECTIVE

2

Finding Surface Areas of Cylinders

If you cut a label from a soup can, you will see that the label is a rectangle. The height of the rectangle is the height of the can. The base length of the rectangle is the circumference of the can.



Key Concepts

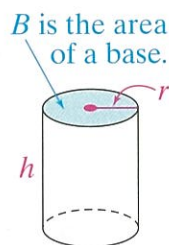
Surface Area of a Cylinder

The lateral area of a cylinder is the product of the circumference of the base and the height of the cylinder.

$$L.A. = 2\pi rh$$

The surface area of a cylinder is the sum of the lateral area and the areas of the two bases.

$$S.A. = L.A. + 2B$$



3

EXAMPLE

Finding Surface Area of a Cylinder

**Packaging** Find the surface area of the can at the right.

**Step 1** Find the lateral area.

$$\begin{aligned}
 L.A. &= 2\pi rh && \text{Use the formula for lateral area.} \\
 &\approx 2(3.14)(3.5)(11.5) && r = 3.5 \text{ and } h = 11.5. \\
 &\approx 253
 \end{aligned}$$

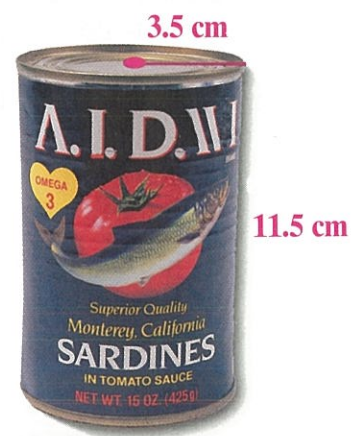
**Step 2** Find the surface area.

$$\begin{aligned}
 S.A. &= L.A. + 2B && \text{Use the formula for surface area.} \\
 &\approx 253 + 2(3.14)(3.5)^2 && L.A. \approx 253 \text{ and } B = \pi(3.5)^2. \\
 &\approx 253 + 77 \\
 &= 330
 \end{aligned}$$

- The surface area of the can is about  $330 \text{ cm}^2$ .

Check Understanding Example 3

- Find the surface area of a can with radius 5 cm and height 20 cm.



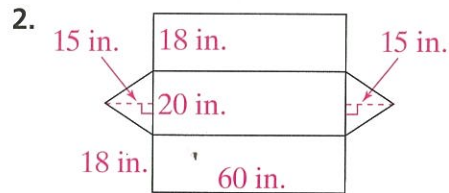
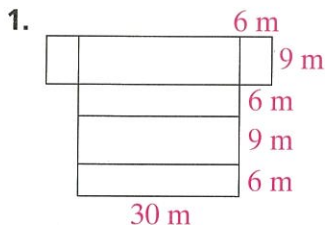
# EXERCISES

For more exercises, see *Extra Practice*.

## Practice and Problem Solving

**A** Practice by Example  
**Example 1**  
 (page 545)

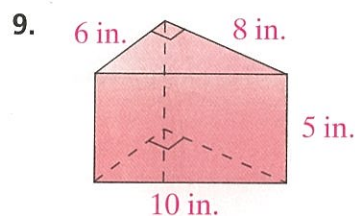
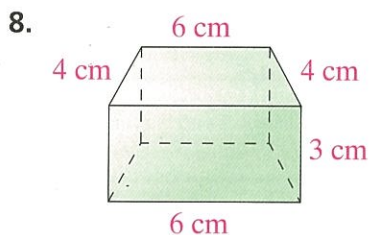
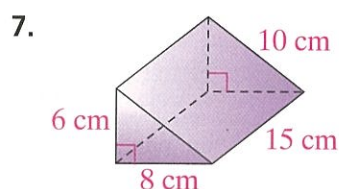
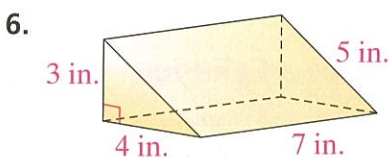
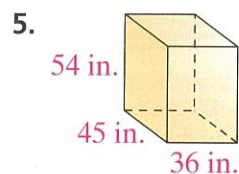
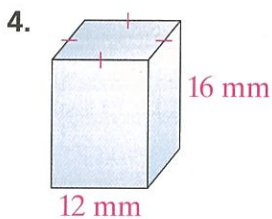
Find, to the nearest square unit, the surface area of the space figure represented by each net.



3. The base of a rectangular prism is 3 in. by 5 in. The height is 11 in.
- Draw and label a net for the prism.
  - Find the surface area of the prism.
  - A similar rectangular prism has dimensions three times the dimensions of the given prism. Find its surface area.
  - How do surface areas of similar prisms compare when dimensions are tripled?

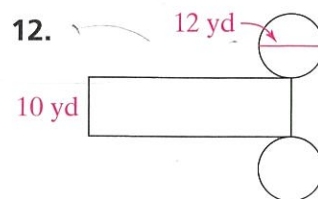
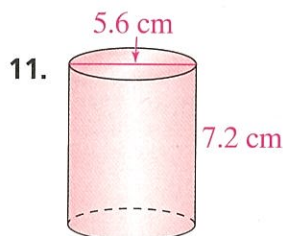
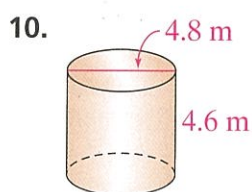
**Example 2**  
 (page 546)

Find the surface area of each prism.



**Example 3**  
 (page 547)

Find the surface area of the cylinder shown or represented. Round to the nearest tenth.

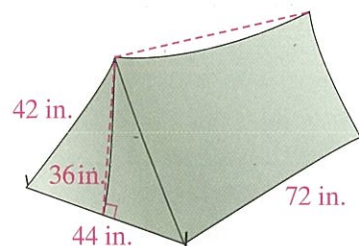


13. **Gift Wrapping** Juliet is trying to wrap a can of mixed nuts that is a birthday gift for her brother. The can has a radius of 8 cm and a height of 10 cm. Approximately how many square centimeters of wrapping paper will cover the gift?

**B Apply Your Skills**

14. A cylinder has radius 8 ft and height 12 ft. Draw and label a net for the cylinder. Find its surface area.
15. Find the surface area of a square prism with base edge 7 m and height 15 m.
16. Find the area of the top and lateral surface of a cylindrical water tank with radius 20 ft and height 30 ft.

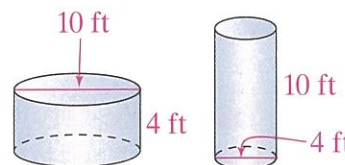
17. **Camping** A tent is approximately the shape of a triangular prism. Approximate the area of the tent, including the bottom, by finding the surface area of the prism.



18. Find the surface area of a cylinder with radius 10 cm and height 8 cm.

19. **Painting** The neighborhood swimming pool needs to be painted. The pool is 40 ft by 60 ft. The depth of the pool is 6 ft throughout.
- How many sides need to be painted?
  - What is the total number of square feet to be painted?
  - The materials for painting the pool cost \$1.50 per square yard. What is the cost of the materials for painting the pool?

20. **Error Analysis** A student explains that the two cylinders at the right have the same surface area. Explain the student's error.

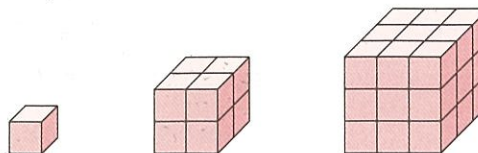


21. **Open-Ended** Describe a real-world situation in which you need to know the surface area of a space figure.
22. **Writing in Math** In a triangular prism, what is the difference between the height of a base and the height of the prism?

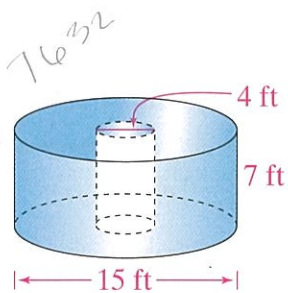
23. **Packaging** You have made two boxes with lids. Which box required more cardboard, a box 8 in. by 6.25 in. by 10.5 in., or a box 9 in. by 5.5 in. by 11.75 in.? Explain.

**C Challenge**

24. **Reasoning** Use the cubes with side lengths of 1, 2, and 3 units.



- Find the surface area of each cube.
- If the length of each side of a cube is doubled, how does that affect the surface area?
- If the length of each side of a cube is tripled, how does that affect the surface area?



25. **Construction** The concrete figure at the left has a hole in it. The surface will be painted except for the inside of the hole. Find the total surface area to be painted to the nearest square foot.
26. **Reasoning** Which has the greater effect on the surface area of a cylinder: doubling the base radius or doubling the height? Justify your answer.

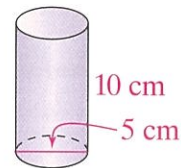
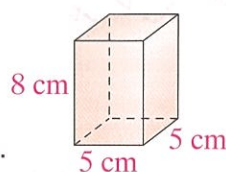


## Test Prep

### Multiple Choice

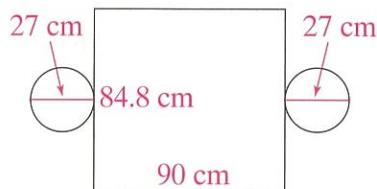
27. Which figure would cost more to paint?

- A. the square prism  
 B. the cylinder  
 C. They would cost the same.  
 D. cannot be determined



28. What is the approximate surface area of the space figure represented by the net?

- F.  $8,777 \text{ cm}^2$   
 G.  $8,333 \text{ cm}^2$   
 H.  $8,202 \text{ cm}^2$   
 I.  $4,387 \text{ cm}^2$



29. Each edge of cube is 40 cm long. What is the surface area of the cube? Use  $2.5 \text{ cm} \approx 1 \text{ in.}$

- A.  $256 \text{ in.}^2$     B.  $1,536 \text{ in.}^2$     C.  $3,840 \text{ in.}^2$     D.  $9,600 \text{ in.}^2$

30. A cylinder has height 10 in. and base radius 2.5 in. What is its lateral area, to the nearest whole unit?

- F.  $235 \text{ in.}^2$     G.  $215 \text{ in.}^2$     H.  $196 \text{ in.}^2$     I.  $157 \text{ in.}^2$



### Test-Taking Tip

When a test question asks for the surface area of a prism, you can sketch the net of the prism to help visualize the entire surface.



### Take It to the NET

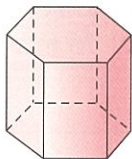
Online lesson quiz at [www.PHSchool.com](http://www.PHSchool.com)

Web Code: ada-1005

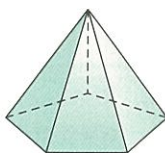
## Mixed Review

### Lesson 10-4 Name each space figure.

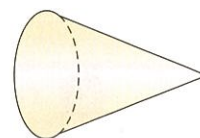
31.



32.



33.



### Lesson 9-9 Graph each point and its image after a reflection over the given line.

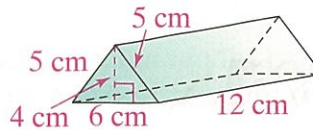
34.  $A(0, 9)$ ;  $x$ -axis    35.  $B(-3, 5)$ ;  $y$ -axis    36.  $C(3, -1)$ ;  $x = 2$

- Lesson 6-2 37. **Recipes** A recipe that serves 6 people calls for  $\frac{1}{2}$  teaspoon of salt. In preparing this recipe for 25 people, about how many teaspoons of salt should you use?



For the example below, follow along with how Diana reads and uses the formulas. Check your understanding by solving the exercises at the bottom of the page.

Find the surface area of the triangular prism at the right.



## What Diana Reads

**Step 1** Find the lateral area.

$$\begin{aligned} \text{L.A.} &= ph \\ &= (5 + 5 + 6)12 \\ &= 192 \end{aligned}$$

**Step 2** Find the surface area.

$$\begin{aligned} \text{S.A.} &= \text{L.A.} + 2B \\ &= 192 + 2\left(\frac{1}{2} \cdot 6 \cdot 4\right) \\ &= 192 + 24 \\ &= 216 \end{aligned}$$

The surface area of the triangular prism is  $216 \text{ cm}^2$ .

## What Diana Thinks

Surface area means lateral area plus base areas.

In the Step 1 formula,

L.A. is the lateral area.

$p$  is the perimeter of the base, so  $p = 5 + 5 + 6$ .

$h$  is the height of the prism, so  $h = 12$ .

Using these values, the Step 1 formula gives  $\text{L.A.} = 192 \text{ cm}^2$ .

In the Step 2 formula,

S.A. is the surface area.

L.A. is the lateral area, 192, from Step 1.

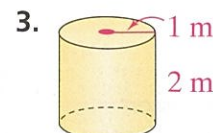
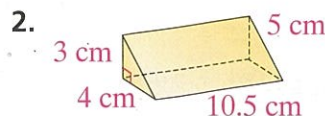
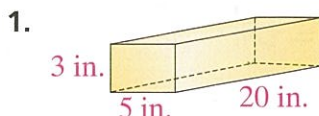
$2B$  is the area of the 2 bases.

Each base is a triangle with area  $B$ . Use the formula for area of a triangle and find  $B = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(6)(4)$ .

Using these values, the Step 2 formula gives  $\text{S.A.} = 216 \text{ cm}^2$ .

## EXERCISES

Write the formulas you use to find the surface area of each figure. Give the meaning and value of each letter or letters in the formula.



# Surface Area: Pyramids, Cones, and Spheres

## OBJECTIVE

1

## Finding Surface Areas of Pyramids

### What You'll Learn

OBJECTIVE

1

To find surface areas of pyramids

OBJECTIVE

2

To find surface areas of cones and spheres

### ... And Why

To find surface areas of real-world objects, such as a basketball



### Check Skills You'll Need

Use the Order of Operations to simplify each expression.

- $\frac{2}{3}(9\pi) + \frac{1}{2}(8\pi)$
- $\frac{3}{4}(12\pi) + \frac{2}{5}(15\pi)$
- $\frac{1}{6}(24\pi) + \frac{1}{3}(3\pi)$
- $\frac{5}{8}(32\pi) + \frac{1}{7}(14\pi)$

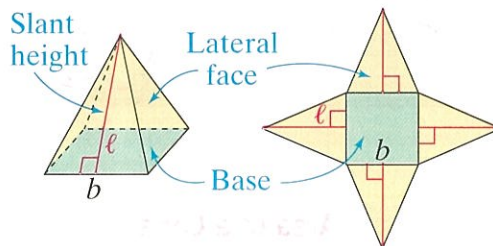


For help, go to Lesson 5-4.

### New Vocabulary

- slant height

In this text, all pyramids are *regular* pyramids. They have regular polygons for bases and congruent isosceles triangles for lateral faces.



You can use the **slant height**  $\ell$ , the height of a face, to find the area of the lateral faces. If  $n$  is the number of lateral triangular faces,

$$\text{L.A. is } n\left(\frac{1}{2}b\ell\right) \text{ or } \frac{1}{2}p\ell.$$

### Key Concepts

### Surface Area of a Pyramid

The lateral area of a pyramid is one half the product of the perimeter of the base and the slant height.

The surface area of a pyramid is the sum of the lateral area and the area of the base.



$$\text{L.A.} = \frac{1}{2}p\ell \quad \text{S.A.} = \text{L.A.} + B$$

1

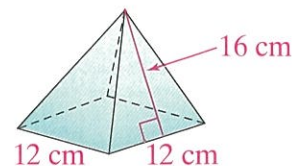
### EXAMPLE

### Finding Surface Area of a Pyramid

Find the surface area of the square pyramid.

**Step 1** Find the lateral area.

$$\begin{aligned} \text{L.A.} &= \frac{1}{2}p\ell && \text{Use the formula for lateral area.} \\ &= \frac{1}{2} \cdot 48 \cdot 16 && p = 4(12) \text{ and } \ell = 16. \\ &= 384 \end{aligned}$$



**Step 1** Find the surface area.

$$\begin{aligned} \text{S.A.} &= \text{L.A.} + B && \text{Use the formula for surface area.} \\ &= 384 + 12^2 && \text{L.A.} = 384 \text{ and } B = 12^2. \\ &= 384 + 144 \\ &= 528 \end{aligned}$$

- The surface area of the pyramid is  $528 \text{ cm}^2$ .



Interactive lesson includes instant self-check, tutorials, and activities.



**Check Understanding Example 1**

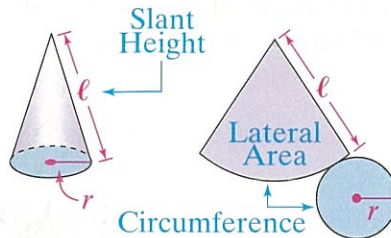
1. A pyramid has a square base with edge 20 ft. The slant height is 8 ft. Find the surface area.

**OBJECTIVE**

**2 Finding Surface Areas of Cones and Spheres**

In this text, every cone is a right circular cone with the vertex of the cone directly over the center of the circular base.

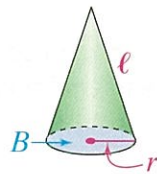
L.A. is  $\frac{1}{2}(2\pi r)\ell$  or  $L.A. = \pi r\ell$ .



**Key Concepts Surface Area of a Cone**

The surface area (S.A.) of a cone is the sum of the lateral area and base area.

$$L.A. = \pi r\ell \quad S.A. = L.A. + B$$

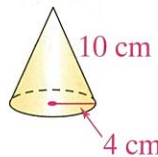


**2 EXAMPLE Finding Surface Area of a Cone**

Find the surface area of the cone at the right.

$$\begin{aligned} L.A. &= \pi r\ell \quad \text{Use the formula for lateral area.} \\ &\approx 3.14(4)(10) \quad r = 4 \text{ and } \ell = 10. \\ &= 125.6 \end{aligned}$$

$$\begin{aligned} S.A. &= L.A. + B \quad \text{Use the formula for surface area.} \\ &\approx 125.6 + 3.14(4)^2 \quad L.A. \approx 125.6 \text{ and } B = \pi(4)^2. \\ &= 125.6 + 50.24 \\ &= 175.84 \end{aligned}$$



- The surface area of the cone is about 176 cm<sup>2</sup>.

**Check Understanding Example 2**

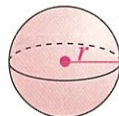
2. A cone has slant height 39 ft and radius 7 ft. Find its surface area.

A sphere has the same area as four circles with the same radius.

**Key Concepts Surface Area of a Sphere**

The surface area of a sphere of radius  $r$  is

$$S.A. = 4\pi r^2.$$





10 in.

### 3 EXAMPLE

### Real-World Problem Solving

**Basketball** Calculate the surface area of a basketball.

$$\begin{aligned} \text{S.A.} &= 4\pi r^2 && \text{Use the formula for surface area.} \\ &\approx 4(3.14)(5)^2 && r = 5 \\ &= 314 \end{aligned}$$

- The surface area of the basketball is about 314 in.<sup>2</sup>.

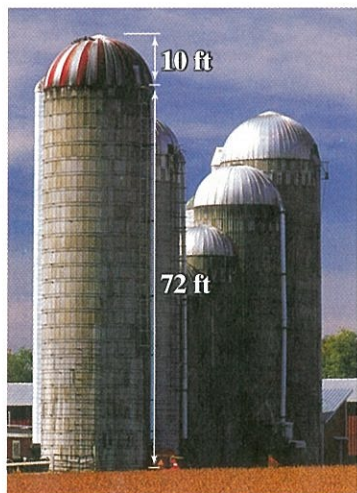
### ✓ Check Understanding Example 3

3. A sphere has a radius of 6 cm. Find its surface area.

You can find the surface area of a space figure that combines two or more figures you have studied.

## More Than One Way

Find the surface area of the silo formed by a half sphere and a cylinder. The diameter of the silo is 20 ft.

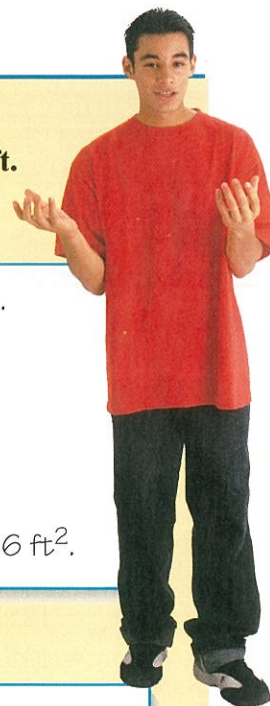


### Roberto's Method

Find the area of each space figure. Then find their sum.

One half sphere	Cylinder
$\text{S.A.} = \frac{1}{2}(4\pi r^2)$	$\text{L.A.} = 2\pi rh$
$\approx \frac{1}{2}(4)(3.14)(10^2)$	$\approx 2(3.14)(10)(72)$
$= 628$	$= 4,521.6$

Surface area of silo is about  $628 + 4,521.6$ , or  $5,149.6 \text{ ft}^2$ .



### Jasmine's Method

Combine formulas before substituting values.

$$\begin{aligned} \text{Surface area of silo} &= \frac{1}{2}\text{S.A. of sphere} + \text{L.A. of cylinder} \\ &= \frac{1}{2}(4\pi r^2) + 2\pi rh \\ &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r(r + h) \\ &\approx 2(3.14)(10)(10 + 72) \\ &= 5,149.6 \end{aligned}$$

Surface area of silo is about  $5,149.6 \text{ ft}^2$ .



### Choose a Method

1. Which method do you prefer? Explain.

# EXERCISES

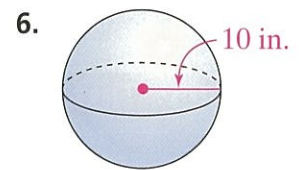
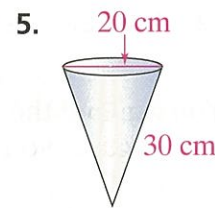
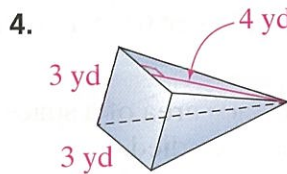
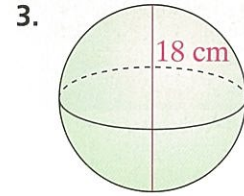
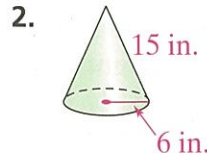
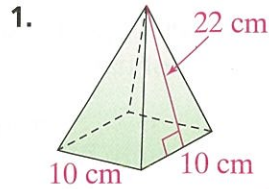
For more exercises, see *Extra Practice*.

## Practice and Problem Solving

### A Practice by Example

Examples 1–3  
(pages 552 and 553)

Find the surface area of each space figure, to the nearest square unit.



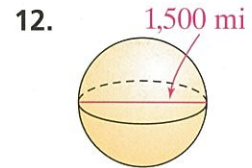
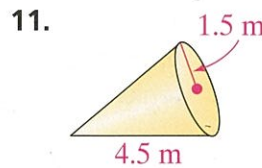
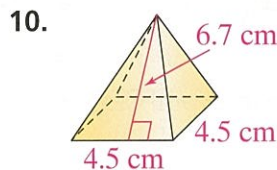
7. The length of the base of a square pyramid is 5 cm. Its slant height is 8 cm. Find the surface area of the square pyramid.

8. The base of a cone has radius 3 ft. Its slant height is 8 ft. Find the surface area of the cone.

9. **Engineering** A spherical ball bearing has a radius of 8 mm. Find the surface area of the ball bearing.

### B Apply Your Skills

Find the surface area of each space figure, to the nearest square unit.



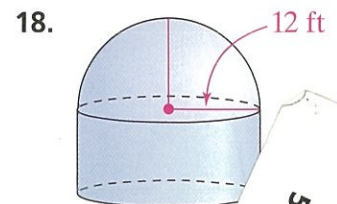
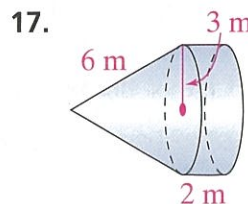
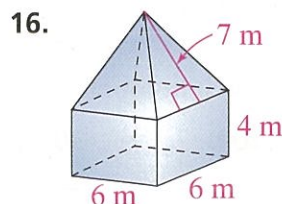
13. **Geography** A globe has a diameter of 18 inches. What is the surface area of the globe to the nearest square inch?

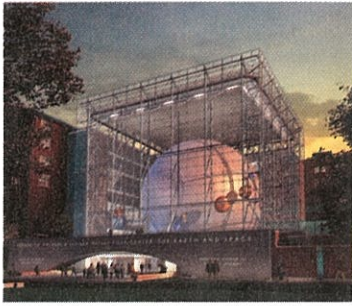
14. **Error Analysis** A friend tells you that the surface area of a square prism with base length 4 m and height 5 m is the same as the surface area of a square pyramid with base length 4 m and height 5 m. Explain your friend's error.

15. **Writing in Math** Write a paragraph explaining how to find the surface area of a cone with slant height 10 in. and base radius 8 in.

### C Challenge

Find the surface area of each figure to the nearest square unit.





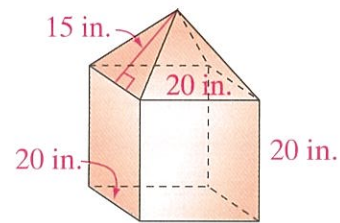
19. **Architecture** The spherical planetarium (left) at the American Museum of Natural History in New York City is 87 ft in diameter.
- What is the surface area of the sphere?
  - The sphere is covered by 2,474 panels to absorb sound. What is the average area of each panel, to the nearest tenth of a square foot?
20. **Reasoning** Which has the greater surface area, a cylinder with height 2 in. and radius of base 2 in., or a sphere with radius 2 in.? Justify your answer.
21. **Geography** Approximately 70% of Earth's surface is covered by water. If the diameter of Earth is approximately 13,000 km, find the approximate surface area *not* covered by water.



## Test Prep

### Multiple Choice

22. What is the surface area of the figure at the right?
- 2,150 in.<sup>2</sup>
  - 2,600 in.<sup>2</sup>
  - 3,000 in.<sup>2</sup>
  - 3,200 in.<sup>2</sup>



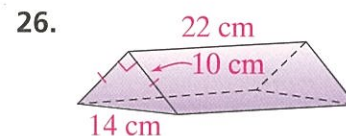
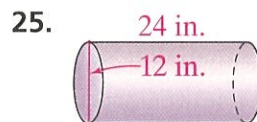
23. What is the ratio of the surface area of a sphere with radius 2 ft to the surface area of a sphere with radius 5 ft?
- 2 to 5
  - 4 to 25
  - 16 to 125
  - 18 to 20

### Extended Response

24. A water storage tank with a roof that is in the shape of a cone has a diameter of 10 ft. The height of the cylindrical part of the tank is 15 ft. The slant height of the roof is 8 ft. (a) What is the radius of the tank? (b) What is the lateral area of the cylindrical part of the tank? (c) What is the surface area of the entire tank?

## Mixed Review

### Lesson 10-5 Find the surface area to the nearest square unit.



- Lesson 8-7 27. Under rate plan A a new computer costs \$200 down and \$20 a month. Under rate plan B the computer costs \$175 down and \$25 a month. After how many months will the amount paid be the same for both plans?

### Lesson 4-1 List all the factors of each number.

28. 21      29. 100      30. 25      31. 32

# Volume: Prisms and Cylinders

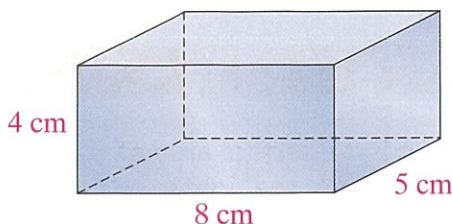
10-7

**OBJECTIVE**

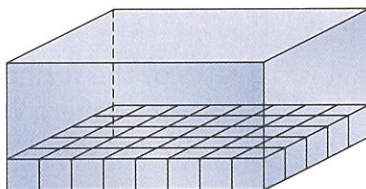
## 1 Finding the Volumes of Prisms

The **volume** of a three-dimensional figure is the number of cubic units needed to fill it. A **cubic unit** is the space occupied by a cube with edges one unit long.

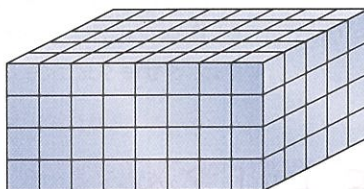
Consider filling the rectangular prism at the right with centimeter cubes.



The bottom layer of the prism contains  $8 \cdot 5 = 40$  centimeter cubes, or a volume of  $40 \text{ cm}^3$  (cubic centimeters).



The prism has four layers of cubes, so it contains  $4 \cdot 40$ , or 160, centimeter cubes in all.



The volume of the prism is  $160 \text{ cm}^3$ .

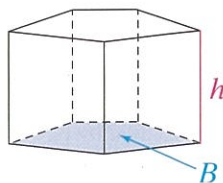
The volume found for the rectangular prism above suggests the following formula.

### Key Concepts

#### Volume of a Prism

The volume  $V$  of a prism is the product of the base area  $B$  and the height  $h$ .

$$V = Bh$$



### What You'll Learn

**OBJECTIVE**

**1**

To find volumes of prisms

**OBJECTIVE**

**2**

To find volumes of cylinders

### ... And Why

To solve real-world problems, such as finding volumes of containers

### Check Skills You'll Need

Find the area of each circle.

- radius = 8 cm
- radius = 12 cm
- diameter = 20 cm

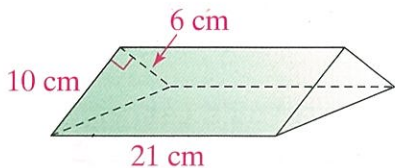
For help, go to Lesson 10-3.

### New Vocabulary

- volume
- cubic unit

**TEXT** Interactive lesson includes instant self-check, tutorials, and activities.

## 1 EXAMPLE Finding Volume of a Prism



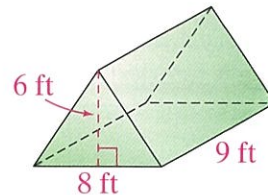
Find the volume of the triangular prism at the left.

$$\begin{aligned} V &= Bh && \text{Use the formula for volume.} \\ &= 30 \cdot 21 && B = \frac{1}{2} \cdot 10 \cdot 6 = 30 \text{ cm}^2 \\ &= 630 && \text{Simplify.} \end{aligned}$$

- The volume is  $630 \text{ cm}^3$ .

### ✓ Check Understanding Example 1

- Find the volume of the triangular prism.



## OBJECTIVE

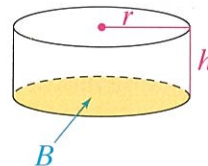
## 2 Finding the Volumes of Cylinders

You can calculate the volume of a cylinder in much the same way that you calculate the volume of a prism.

### Key Concepts Volume of a Cylinder

The volume  $V$  of a cylinder is the base area  $B$  times the height  $h$ .

$$V = Bh$$



## 2 EXAMPLE Real-World Problem Solving

**Packaging** Find the volume of the juice can to the nearest cubic centimeter.

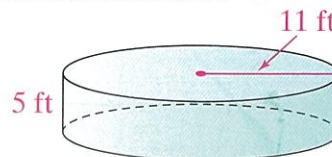
$$\begin{aligned} V &= Bh && \text{Use the formula for volume.} \\ V &= \pi r^2 h && B = \pi r^2 \\ &\approx 3.14 \cdot 3.4^2 \cdot 12 && \text{Replace } r \text{ with } 3.4, \\ &= 435.5808 && \text{and } h \text{ with } 12. \\ &&& \text{Simplify.} \end{aligned}$$

- The volume is about  $436 \text{ cm}^3$ .



### ✓ Check Understanding Example 2

- Find the volume of the cylinder to the nearest cubic foot.
  - How does the volume of this cylinder compare with one having twice its dimensions?



# EXERCISES

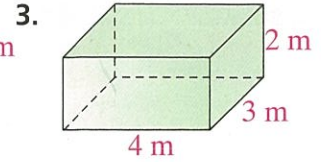
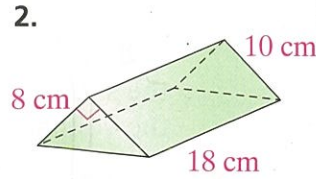
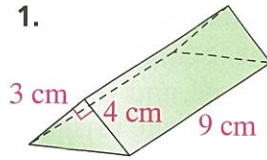
For more exercises, see *Extra Practice*.

## Practice and Problem Solving

### A Practice by Example

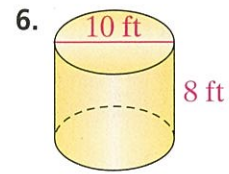
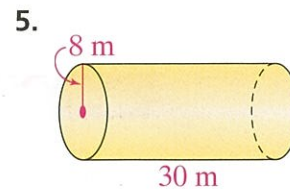
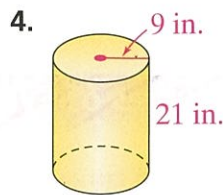
Find the volume of each prism.

**Example 1**  
(page 558)



**Example 2**  
(page 558)

For Exercises 4–7, find the volume of each cylinder to the nearest cubic unit.

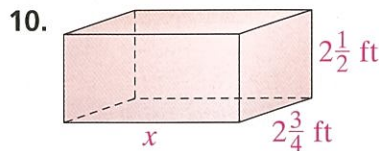


7. a. a mailing tube 25 in. long with a diameter of 4 in.  
b. a mailing tube with double the dimensions in part (a)  
c. How do the volumes of the two mailing tubes compare?

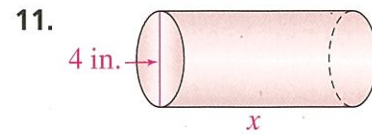
### B Apply Your Skills

8. **Firewood** Wood for a fireplace is often sold by the cord. A cord is 8 ft by 4 ft by 4 ft. How many cubic feet are in a cord of wood?
9. **Storage** An under-the-bed storage box measures 24 in. by 12 in. by 3 in. Find its volume to the nearest cubic centimeter (1 in. = 2.54 cm).

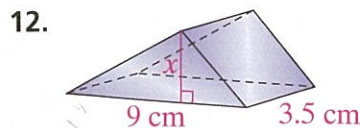
Find each missing dimension. Use  $\pi \approx 3.14$ .



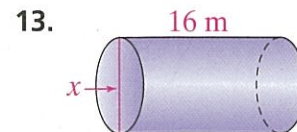
$V = 38.5 \text{ ft}^3$   
Length = ■



$V = 125.6 \text{ in.}^3$   
Height  $\approx$  ■



$V = 50.4 \text{ cm}^3$   
Height of triangle = ■

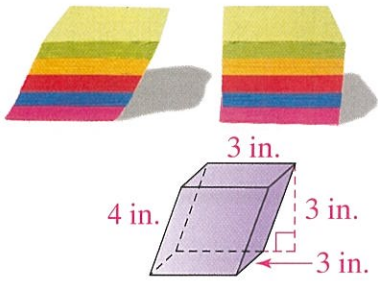


$V = 1,256 \text{ m}^3$   
Diameter  $\approx$  ■

**Writing in Math** Describe one real object with the given shape. Explain why you might want to find the volume of the object.

14. triangular prism    15. cylinder    16. rectangular prism

- C Challenge** 17. **Construction** Concrete is sold by the yard, which means by the cubic yard. It costs \$70 per yard. How many cubic feet are in a cubic yard? How much would it cost to pour a slab that is 14 ft by 16 ft by 6 in. for a patio?



18. **Reasoning** The two stacks of paper in the photo at the left contain the same number of sheets. The first stack forms an oblique prism; the second forms a right prism. The stacks have the same height, base, and volume. Use this information to find the volume of the oblique prism shown below the stacks.

19. **Error Analysis** A student explains that a cylinder with radius 1 in. and height 3 in. has half the volume of one with radius 2 in. and height 3 in. Explain the student's error.



## Test Prep

### Multiple Choice

20. One side of a triangle is 5 cm and its corresponding height is 15 cm. The triangle is the base of a triangular prism with height 5 cm. What is the volume of the prism?  
 A.  $12.5 \text{ cm}^3$    B.  $37.5 \text{ cm}^3$    C.  $187.5 \text{ cm}^3$    D.  $375.0 \text{ cm}^3$
21. What is the height of a rectangular prism with length 5 ft, width 2 ft, and volume  $120 \text{ ft}^3$ ?  
 F. 12 ft   G. 10 ft   H. 6 ft   I. 2 ft

### Extended Response

22. A manufacturer is deciding whether to package table salt in a cylinder or in a rectangular prism made of cardboard. The cylinder has radius 4 cm and height 13.5 cm. The prism is 7 cm by 7 cm by 13.5 cm.
- What volume of salt will each package hold?
  - Each package will use how many square centimeters of cardboard?
  - Which seems to be the better type of package? Explain.



### Take It to the NET

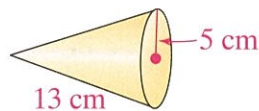
Online lesson quiz at  
[www.PHSchool.com](http://www.PHSchool.com)  
 Web Code: ada-1007

## Mixed Review

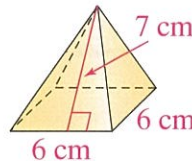
### Lesson 10-6

Find the surface area of each figure, to the nearest square unit.

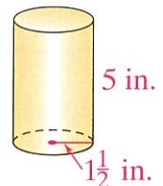
23.



24.



25.

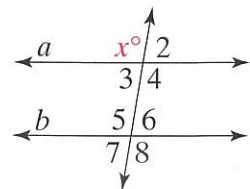


### Lesson 9-2

26. If  $a \parallel b$  in the figure at the right, find the measures of angles 2–8 in terms of  $x$ .

### Lesson 5-6

27. **Money** Juan has \$3.80 in coins. He has 6 quarters and 12 dimes. The rest are nickels. How many nickels does he have?



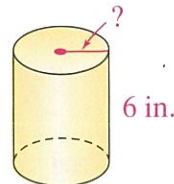




A calculator can help you avoid a *rounding error*. However, you have to know how to work with exact values in unsimplified form.

### EXAMPLE

Find the volume of a cylinder with height 6 and circumference 15. Round to the nearest tenth.



#### Method 1

$$C = 2\pi r$$

$$15 = 2\pi r$$

$$\frac{15}{2\pi} = r$$

$$2.4 \approx r$$

$$V = Bh$$

$$= \pi r^2 h$$

$$\approx \pi(2.4)^2(6)$$

$$\approx 108.6$$

formula for circumference

$$C = 15$$

Divide by  $2\pi$ .

Simplify (at left).

formula for volume

$$B = \pi r^2$$

Replace  $r$  and  $h$ .

Use a calculator.

#### Method 2

$$C = 2\pi r$$

$$15 = 2\pi r$$

$$\frac{15}{2\pi} = r$$

$$V = Bh$$

$$= \pi r^2 h$$

$$= \pi\left(\frac{15}{2\pi}\right)^2(6)$$

$$\approx 107.4$$

In Method 1, the intermediate value  $r \approx 2.4$  gives a final result that has a greater *rounding error*. In Method 2, note that the formula for volume uses the exact value of the radius,  $\frac{15}{2\pi}$ .

## EXERCISES

**What intermediate value must you find to solve the problem? Give its exact value and its approximate value to the nearest tenth.**

1. Find the area of a circle with circumference 12.
2. Find the volume of a cylinder with radius 3 and lateral area 10.

**Solve each problem to the nearest tenth by two methods. In one method, use an approximate intermediate value. In the other method, use the exact intermediate value.**

3. Find the surface area of a cone with slant height 5 and lateral area 9.
4. Find the surface area of a sphere with circumference 13.
5. Find the surface area and volume of a cylinder with lateral area 10 and height 8.
6. **Writing in Math** In the Example, which method do you think makes better use of the calculator? Explain.

## 10-8

## Problem Solving

## Make a Model

## What You'll Learn

OBJECTIVE

1

To make a model

## ... And Why

To build the largest box possible from a given rectangle

## ✓ Check Skills You'll Need

Draw each figure described below.

1. a rectangle with small squares drawn in each corner
2. a rectangle divided into eight congruent rectangles
3. two parallelograms that have different shapes but the same perimeter

For help, go to Lesson 9-3.

## OBJECTIVE

1

## Make a Model

**Math Strategies in Action** Architects build and use models when they plan. When they design buildings, they experiment with models. When they design packaging, they first create prototype models.



## 1 EXAMPLE

## Real-World Problem Solving

**Packaging** A box company makes boxes to hold popcorn. Each box is made by cutting the square corners out of a rectangular sheet of cardboard. The rectangle is  $8\frac{1}{2}$  in. by 11 in. What are the dimensions of the box that will hold the most popcorn if the square corners have side lengths 1 in., 2 in., 3 in., and 4 in.?

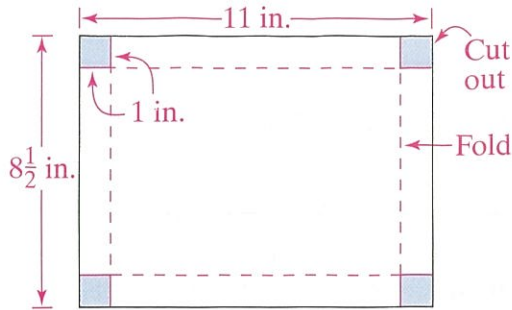
## Read and Understand

1. What is the goal of the problem?
2. What information do you have to help you build a model?

## Plan and Solve

To find the size that will hold the greatest amount of popcorn, you must find the dimensions that will give you the greatest volume.

Build four boxes using sheets of  $8\frac{1}{2}$  in.-by-11 in. paper. Test four whole-number lengths of cuts.



3. a. What are the dimensions of the box with corners 1 in. by 1 in.?  
b. What is the volume of this box?
4. When you cut a 2 in.-by-2 in. square from each corner, what effect does that have on the length, width, and height of the box?

Measure to find the dimensions of each of your boxes. Then find the volume of each box.

5. Which box has the greatest volume?
6. Is it possible to create a box that has 5 in.-by-5 in. corners? Explain.

### Look Back and Check

A table is another way to organize your information and solve the problem.

7. List the size of the cut, and then find the length, width, and height of the box. Find each volume.

Size of Cut	Length	Width	Height	Volume
1 in.	9 in.	6.5 in.	1 in.	$58.5 \text{ in.}^3$
2 in.	■ in.	■ in.	■ in.	■ $\text{in.}^3$
3 in.	■ in.	■ in.	■ in.	■ $\text{in.}^3$
4 in.	■ in.	■ in.	■ in.	■ $\text{in.}^3$

### Check Understanding

8. a. Use a table to find the volume of a box folded from an  $8\frac{1}{2}$  in.-by-11 in. sheet of paper if the square corners are  $1\frac{1}{2}$  in.;  $2\frac{1}{2}$  in.;  $3\frac{1}{2}$  in.  
b. Did you find dimensions of a box that holds a greater volume than you did in Question 7? Which dimensions are they?

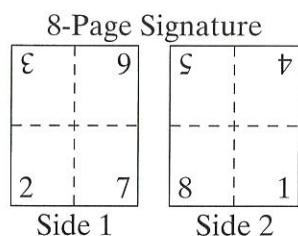
# EXERCISES

For more exercises, see *Extra Practice*.

## Practice and Problem Solving

### A Practice by Example Solve by making a model.

**Example 1**  
(page 562)



- You cut square corners off a piece of cardboard with dimensions 16 in. by 20 in. You then fold the cardboard to create a box with no lid. To the nearest inch, what dimensions will give you the greatest volume?
- Publishing** Newspapers, books, and magazines often are printed in groups of 8, 16, or 32 pages, called *signatures*. The diagram at the left shows how pages should be positioned for an 8-page signature. The pages are positioned to print on both sides of the paper that is fed through the printing press. When the paper is folded, the pages are in order. Make a model to show one way to position the pages in a 16-page book.

### B Apply Your Skills Solve using any strategy.

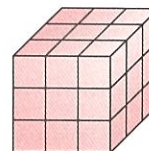
#### Strategies

- Account for All Possibilities
- Draw a Diagram
- Look for a Pattern
- Make a Model
- Make a Table
- Simplify the Problem
- Simulate the Problem
- Solve by Graphing
- Try, Test, Revise
- Use Multiple Strategies
- Work Backward
- Write an Equation
- Write a Proportion

- The length of a rectangle is twice its width. The perimeter of the rectangle is 90 cm. What are the length and width?
- Packaging** A company packages snack mix in cylindrical tubes. Each tube is made from a rectangle of cardboard. The bases of the cylinder are plastic. The cardboard comes in  $8\frac{1}{2}$  in.-by-11 in. sheets. To hold the greatest amount of mix, should the longer side or shorter side be the height? Justify your answer.
- Pets** A dog owner wants to use 200 ft of fencing to enclose the greatest possible area for his dog. He wants the fenced area to be rectangular. What dimensions should he use?
- Writing in Math** You want to find how the length of a pendulum affects the time the pendulum takes to swing back and forth. Explain how you would model the situation.
- An alphabet book will have one letter on each page. Eight pages will be printed on a single piece of paper. You fold the paper in half. Then you fold it in half again and trim the edges. The eight pages appear in order. Draw the layout for two sides of the large sheet for the letters A–H. Explain why two layouts are possible.

### C Challenge

- Reasoning** In the parts (a)–(c), how many of the indicated cubes are there in this 3-by-3-by-3 cube?
  - 1-by-1-by-1 cubes
  - 2-by-2-by-2-cubes
  - 3-by-3-by-3 cubes
  - How many 3-by-3-by-3 cubes would be in a 5-by-5-by-5 cube?

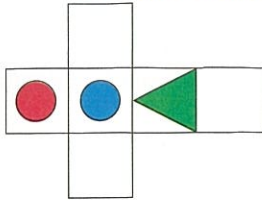


- One base of a trapezoid is twice as long as the other base. The height is the same as the shorter base. If the area is  $24 \text{ cm}^2$ , what is the height of the trapezoid?

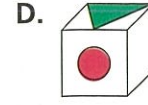
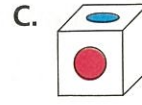
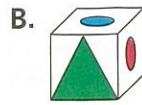


## Test Prep

### Multiple Choice



10. Which space figure below can you fold from the net at the left?



11. A rectangle has a perimeter of 24. The length and width are integers. For the least possible area, what are its dimensions?

- F. 1 by 11    G. 1 by 24    H. 2 by 4    I. 6 by 6

12. A pyramid and a rectangular prism have the same base and height. The volume of the rectangular prism is how many times the volume of the pyramid?

- A.  $\frac{1}{3}$     B.  $\frac{1}{2}$     C. 2    D. 3



### Take It to the NET

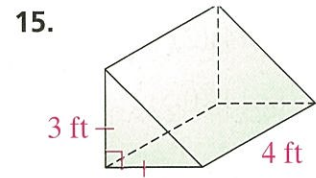
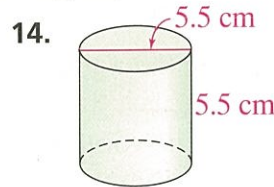
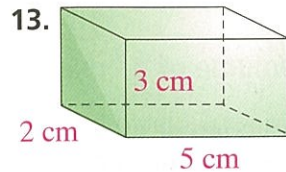
Online lesson quiz at  
[www.PHSchool.com](http://www.PHSchool.com)

Web Code: ada-1008

## Mixed Review

### Lesson 10-7

Find the volume of each figure, to the nearest tenth.



### Lesson 8-2

Find the solutions of each equation when  $x$  is 0, 1, and  $-1$ .

16.  $2x - y = 10$

17.  $5x + y = 15$

18.  $2x + 3y = 6$



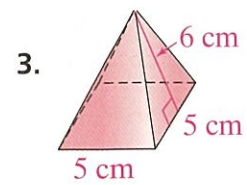
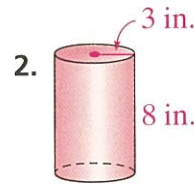
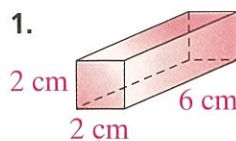
## Checkpoint Quiz 2

## Lessons 10-4 through 10-8

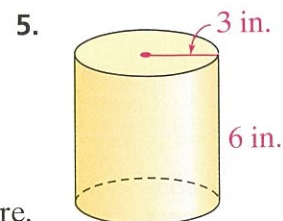
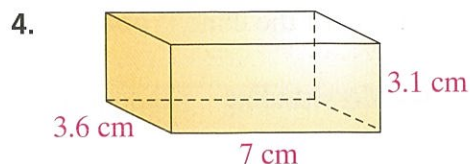


Instant self-check  
quiz online and  
on CD-ROM

Name each space figure. Find its surface area, to the nearest square unit.



Find the volume of each figure to the nearest tenth.



6. **Open-Ended** Choose a space figure. Draw its net.

# Volume: Pyramids, Cones, and Spheres

## What You'll Learn

**OBJECTIVE 1** To find volumes of pyramids and cones

**OBJECTIVE 2** To find volumes of spheres

## ... And Why

To find out how much water is displaced by a space figure

## Check Skills You'll Need

### Multiply.

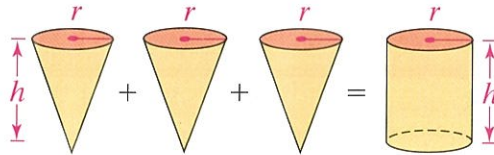
- $\frac{1}{3}(3.14)(2)^2(5)$
- $\frac{1}{3}(4)^2(6)$
- $\frac{4}{3}(3.14)(2)^3$
- $\frac{4}{3}(3.14)(0.5)^3$

For help, go to Lesson 5-4.

## OBJECTIVE

### 1 Finding Volumes of Cones and Pyramids

You can fill three cones with sand and pour the contents into a cylinder with the same height and radius. You will fill the cylinder evenly to the top.



The volume of the cone is one third the volume of the cylinder.

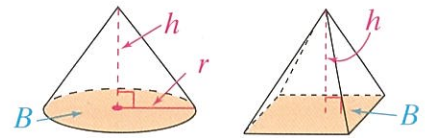
The same relationship is true of a pyramid and a prism with the same base and height.

## Key Concepts

### Volume of a Cone and of a Pyramid

The volume  $V$  of a cone or a pyramid is  $\frac{1}{3}$  the product of the base area  $B$  and the height  $h$ .

$$V = \frac{1}{3}Bh$$



### 1 EXAMPLE Finding Volume of a Cone

**Find the volume of the cone.**

$$V = \frac{1}{3}Bh$$

Use the formula for volume.

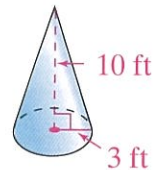
$$V = \frac{1}{3}\pi r^2 h$$

$$B = \pi r^2$$

$$\approx \frac{1}{3}(3.14)(3)^2(10)$$

Replace  $r$  with 3 and  $h$  with 10. Simplify.

$$= 94.2$$



- The volume of the cone is about  $94 \text{ ft}^3$ .

### Check Understanding Example 1

- Find the volume, to the nearest cubic unit, of a cone with height 5 cm and radius of base 2 cm.

**TEXT** Interactive lesson includes instant self-check, tutorials, and activities.

## 2 EXAMPLE Finding Volume of a Pyramid

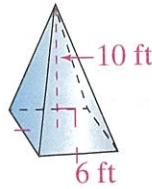
Find the volume of the square pyramid.

$$V = \frac{1}{3}Bh \quad \text{Use the formula for volume.}$$

$$V = \frac{1}{3}s^2h \quad B = s^2$$

$$= \frac{1}{3}(6)^2(10) \quad \text{Replace } s \text{ with } 6 \text{ and } h \text{ with } 10.$$

$$= 120 \quad \text{Simplify.}$$



- The volume of the pyramid is  $120 \text{ ft}^3$ .

### ✓ Check Understanding Example 2

- Find the volume of a square pyramid that has a side of 5 ft and a height of 20 ft.

## OBJECTIVE

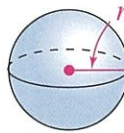
## 2 Finding Volumes of Spheres

Below is the formula for the volume of a sphere.

### Key Concepts Volume of a Sphere

The volume  $V$  of a sphere with radius  $r$  is  $\frac{4}{3}\pi$  times the cube of the radius.

$$V = \frac{4}{3}\pi r^3$$



## 3 EXAMPLE Real-World Problem Solving

**Snow Spheres** You build a snow statue with snow spheres. What is the volume of the snow in the bottom sphere?

$$V = \frac{4}{3}\pi r^3 \quad \text{Use the volume formula.}$$

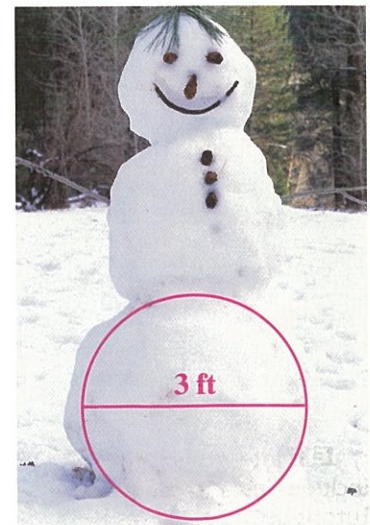
$$\approx \frac{4}{3}(3.14)(1.5)^3 \quad \text{Replace } r \text{ with } 1.5.$$

$$= 14.13 \quad \text{Simplify.}$$

- The volume of the bottom snow sphere is about  $14 \text{ ft}^3$ .

### ✓ Check Understanding Example 3

- Find the volume of each sphere to the nearest whole number.
  - radius = 15 m
  - diameter = 7 mi



# EXERCISES

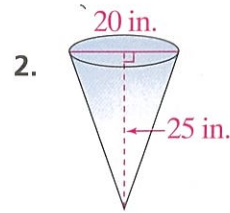
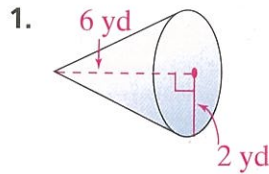
For more exercises, see *Extra Practice*.

## Practice and Problem Solving

### A Practice by Example

In Exercises 1–4, find the volume of each cone to the nearest cubic unit.

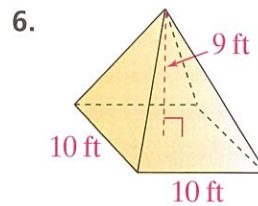
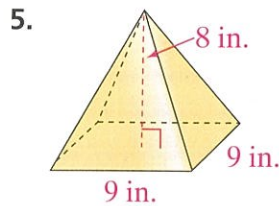
**Example 1**  
(page 566)



3. height 12 cm, radius 21 cm      4. height 7 in., radius 7 in.

**Example 2**  
(page 567)

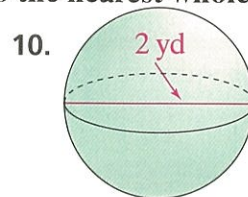
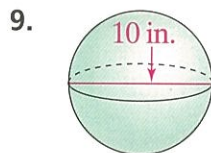
In Exercises 5–8, find the volume of each square pyramid.



7. edge 3.2 cm, height 6 cm      8. edge 90 mm, height 300 mm

**Example 3**  
(page 567)

Find the volume of each sphere to the nearest whole number.



11.  $r = 12$  cm    12.  $r = 3.5$  in.    13.  $d = 60$  yd    14.  $d = 30$  m

### B Apply Your Skills

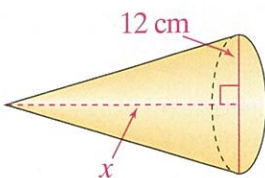
15. **Plants** A cone-shaped paper cup is 7 cm high with a diameter of 6 cm. If the ivy plant on Julia's desk needs 240 mL of water, about how many paper cups of water will she use to water it? ( $1 \text{ mL} = 1 \text{ cm}^3$ )
16. **Snacks** How much frozen yogurt can you pack inside a cone that is 5 in. high with a radius of 1.25 in.?
17. **Packaging** Tennis balls with a diameter of 2.5 in. are sold in cans of three (left). The can is a cylinder. What is the volume of the space in the can not occupied by tennis balls? Assume the balls touch the can on the sides, top, and bottom.
18. **Writing in Math** Explain how you remember formulas for finding volumes of prisms and pyramids, cylinders and cones, and spheres.
19. **Physics** You place a steel ball with diameter 4 cm in a water-filled cylinder that is 5 cm in diameter and 10 cm high. What volume of water will spill out of the cylinder?





Find the missing dimension. Round to the nearest unit.

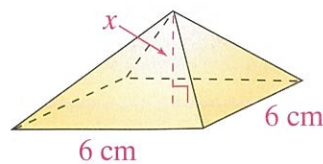
20.



$$V = 819 \text{ cm}^3$$

$$\text{Height} \approx \blacksquare$$

21.



$$V = 38 \text{ cm}^3$$

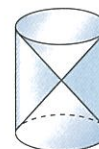
$$\text{Height} = \blacksquare$$

22. Sphere: volume =  $36\pi \text{ yd}^3$ , diameter =  $\blacksquare$

23. Cone: volume =  $424 \text{ m}^3$ , diameter = 18 m, height  $\approx \blacksquare$

**C Challenge**

24. You want to fill the top part of an hourglass  $\frac{2}{3}$  full of salt. The height of the hourglass is 20 cm, and the radius of the base is 8 cm. Find the volume of salt needed.



25. **Error Analysis** A student tells a class that if you double the radius of a sphere, the volume will be multiplied by 6. Explain the student's error.



**Test Prep**

**Gridded Response**

26. The eight segments from the center of a cube to the eight corners of the cube form the edges of six pyramids. If one edge of the cube is 4 in., what is the volume of each pyramid, to the nearest cubic inch?

27. What is the volume of a square pyramid with height 3.25 yd and base edge 5.5 yd, to the nearest tenth of a cubic yard?

28. What is the volume of a sphere with diameter 5.5 m, to the nearest cubic meter?

29. What is the volume of a cone with radius 4.8 cm and height 12 cm, to the nearest tenth of a cubic centimeter? Use 3.14 for  $\pi$ .

30. How many cones of radius 1 m and height 1 m have total volume equal to the volume of a sphere with radius 1 m?



**Take It to the NET**

Online lesson quiz at  
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**Mixed Review**

**Lesson 7-5**

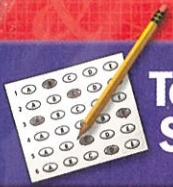
Solve each equation.

31.  $\frac{5}{6}x = \frac{1}{6}x + 12$     32.  $5y + 2 = 3y - 10$     33.  $3a + 10 = 12 - 2a$

**Lesson 5-9**

Simplify each expression.

34.  $(3ab^2)^3$     35.  $-(4x)^2$     36.  $(-2p^2)^4$     37.  $\left(-\frac{3}{8}\right)^2$     38.  $\left(\frac{2x}{y^3}\right)^2$



Some multiple-choice questions cannot be answered because there is insufficient information or there is more than one possible answer. If so, then one of the answer choices will be "cannot be determined."

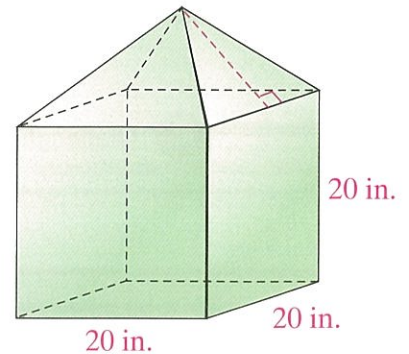
1 EXAMPLE

What is the surface area of the figure at the right?

- A. 2,600 in.² B. 3,200 in.² C. 8,600 in.² D. cannot be determined

The bottom part is a cube. You know its length, width, and height, so you can find its base area and its lateral area. The top part is a pyramid. You know the perimeter of the base, but you need to know the slant height of its faces to find its lateral area.

- There is insufficient information. The correct choice is D.



2 EXAMPLE

A rectangular prism with height 5 cm has volume 60 cm³. What is the perimeter of the base?

- F. 14 cm G. 16 cm H. 26 cm I. cannot be determined

You know  $V = Bh$ .

$60 = B(5)$       Replace  $V$  with 60 and  $h$  with 5.

$12 = B$       Solve for  $B$ .

For  $B = 12 \text{ cm}^2$ , the rectangular base can be 1 cm by 12 cm, 2 cm by 6 cm, or 3 cm by 4 cm. The perimeter can be 26 cm (choice H), 16 cm (choice G), or 14 cm (choice A).

- There is more than one possible answer. The correct choice is I.

EXERCISES

If the answer to an exercise cannot be determined, choose "cannot be determined" and explain your reasoning.

- 1. The base of a square prism has sides of 14 in. What is the surface area of the prism?
A. 56 in.² B. 196 in.² C. 784 in.² D. cannot be determined
2. A trapezoid has height 8 and area 28. What are the lengths of its bases?
F. 6 and 1 G. 5 and 2 H. 4 and 3 I. cannot be determined

## Vocabulary

altitude (p. 523)  
altitude of a triangle (p. 527)  
area (p. 522)  
cone (p. 539)  
cubic unit (p. 557)

cylinder (p. 539)  
lateral area (p. 546)  
net (p. 540)  
prism (p. 539)  
pyramid (p. 539)

slant height (p. 552)  
space figure (p. 539)  
sphere (p. 539)  
surface area (p. 545)  
volume (p. 557)



Choose the vocabulary term that correctly completes the sentence.

- The sum of the areas of the lateral faces of a prism is the ? of a prism.
- A ? is a space figure with one circular base and one vertex.
- The ? of a parallelogram is a line segment drawn from the side opposite the base to the base, that is perpendicular to the base.
- The ? of a three-dimensional figure is the number of cubic units needed to fill it.
- The set of all points in space that are a given distance from a given point called the center is a ?.
- A ? has two parallel bases that are congruent circles.
- The ? of a figure is the number of square units it encloses.



### Take It to the NET

Online vocabulary quiz at [www.PHSchool.com](http://www.PHSchool.com)

Web Code: adj-1051

## Skills and Concepts

### 10-1 and 10-2 Objectives

- ▼ To find areas of rectangles (p. 522)
- ▼ To find areas of parallelograms (p. 523)
- ▼ To find areas of triangles (p. 527)
- ▼ To find areas of trapezoids (p. 528)

The **area** of a polygon is the number of square units enclosed by the polygon. To find the areas of parallelograms, triangles, or trapezoids, use the appropriate formulas.

parallelogram

triangle

trapezoid

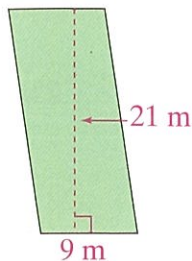
$$A = bh$$

$$A = \frac{1}{2}bh$$

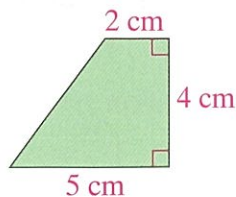
$$A = \frac{1}{2}h(b_1 + b_2)$$

Find the area of the shaded region in each figure.

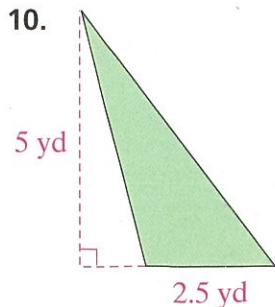
8. Parallelogram



9. Trapezoid



10.

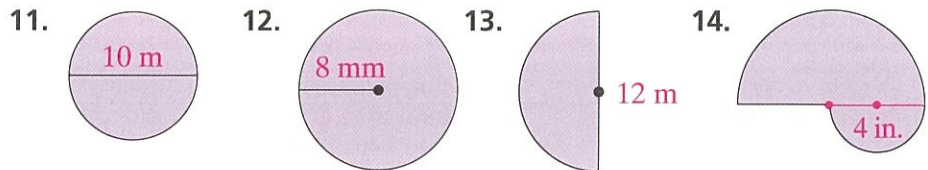


### 10-3 Objectives

- ▼ To find areas of circles (p. 532)
- ▼ To find areas of irregular figures that include parts of circles (p. 534)

To find the area of a circle, use the formula  $A = \pi r^2$ . Use 3.14 for  $\pi$ .

Find the area of each figure to the nearest square unit.

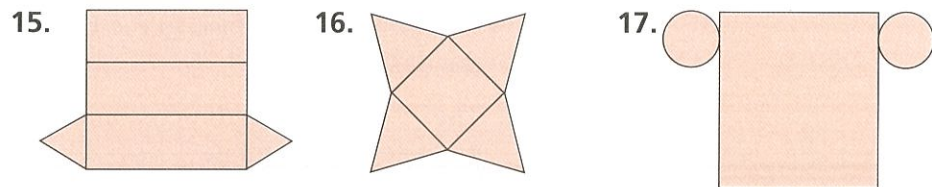


### 10-4 Objectives

- ▼ To identify common space figures (p. 539)
- ▼ To identify space figures from nets (p. 540)

Name **pyramids** and **prisms** by the shapes of their bases. A **cylinder** is a space figure with two circular bases. **Cones** have one circular base and one vertex. **Nets** are flat patterns for space figures.

Name the space figure represented by each net.



### 10-5 Objectives

- ▼ To find surface areas of prisms (p. 545)
- ▼ To find surface areas of cylinders (p. 547)

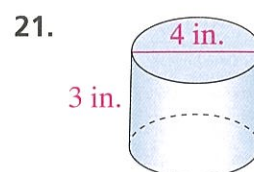
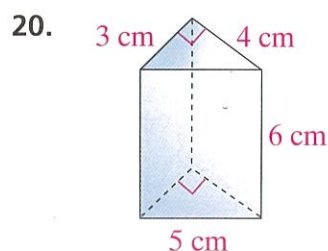
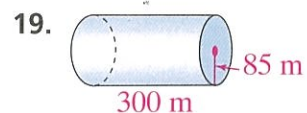
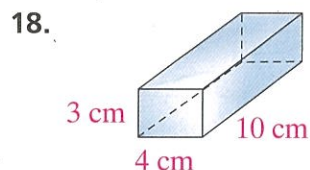
The **lateral area** of a prism is the sum of the areas of the lateral faces. The lateral area of a cylinder is the area of the curved surface. The **surface area** of a prism or a cylinder is the sum of the lateral area and the areas of the two bases.

To find surface area, use the appropriate formula.

$$\begin{aligned} \text{prism} \\ \text{L.A.} &= ph \\ \text{S.A.} &= \text{L.A.} + 2B \end{aligned}$$

$$\begin{aligned} \text{cylinder} \\ \text{L.A.} &= 2\pi rh \\ \text{S.A.} &= \text{L.A.} + 2B \end{aligned}$$

Find the surface area to the nearest square unit. Use 3.14 for  $\pi$ .



### 10-6 Objectives

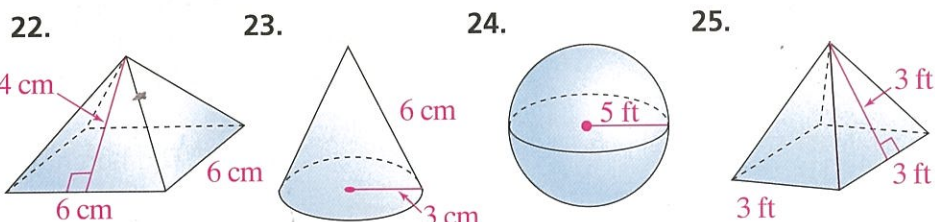
- ▼ To find surface areas of pyramids (p. 552)
- ▼ To find surface areas of cones and spheres (p. 553)

For pyramids and cones, use **slant height**  $\ell$  to find the lateral area. For a regular pyramid, if  $n$  is the number of lateral faces, you can find the area of one face and then multiply by  $n$ . The lateral area of a cone is the area of the curved surface. The surface area of a pyramid or a cone is the sum of the lateral area and the base area.

To find surface area, use the appropriate formula.

pyramid	cone	sphere
$L.A. = n\left(\frac{1}{2}b\ell\right)$	$L.A. = \pi r\ell$	$S.A. = 4\pi r^2$
$S.A. = L.A. + B$	$S.A. = L.A. + B$	

**Find the surface area of each figure, to the nearest square unit.**



### 10-8 Objective

- ▼ To make a model (p. 562)

To solve some problems, make a model.

26. A gift box measures 6 in. along each edge. You cut a rectangular sheet of wrapping paper to get a single piece with which you can cover the box without overlapping. What are the smallest possible dimensions of the original sheet of wrapping paper?
27. A 12 m-by-15 m rectangular garden has a walk 1 m wide around it. Describe how you would find the area of the walk.

### 10-7 and 10-9 Objectives

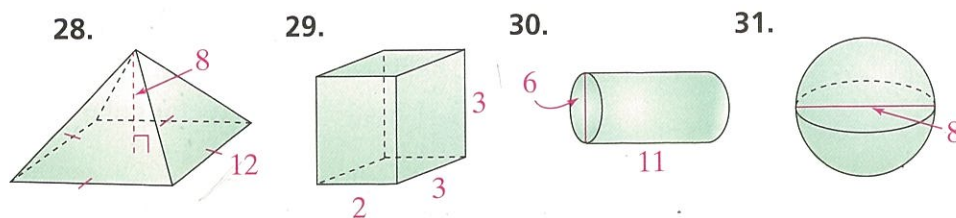
- ▼ To find volumes of prisms (p. 557)
- ▼ To find volumes of cylinders (p. 558)
- ▼ To find volumes of cones and pyramids (p. 566)
- ▼ To find volumes of spheres (p. 567)

**Volume** is the measure of how much a space figure can hold.

To find volume, use the appropriate formula.

prisms and cylinders	pyramids and cones	spheres
$V = Bh$	$V = \frac{1}{3}Bh$	$V = \frac{4}{3}\pi r^3$

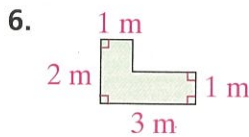
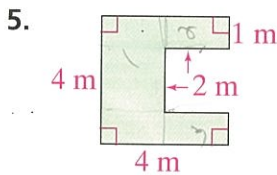
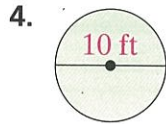
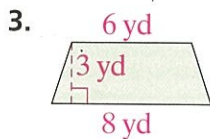
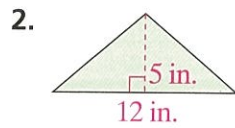
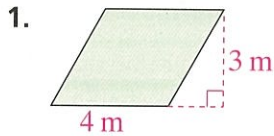
**Find each volume to the nearest cubic unit.**





Use 3.14 for  $\pi$  as needed on this page.

Find the area of each figure.

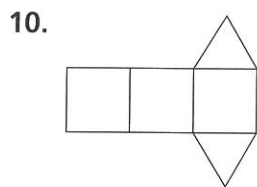
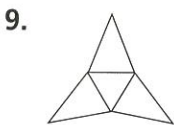


Find the missing measures.

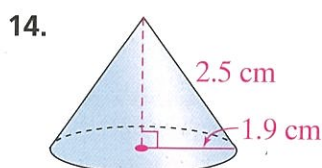
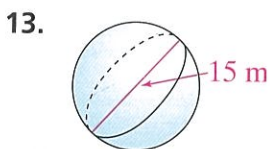
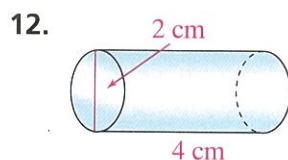
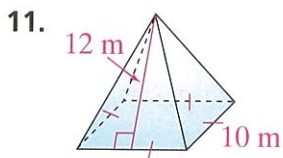
7. circle  
 $d = 4$  cm  
 $A = \blacksquare$  cm<sup>2</sup>

8. triangle  
 $b = 7$  m  
 $h = 4$  m  
 $A = \blacksquare$  m<sup>2</sup>

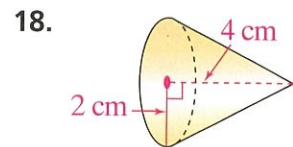
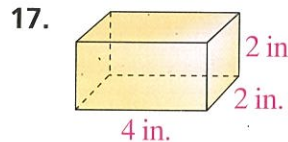
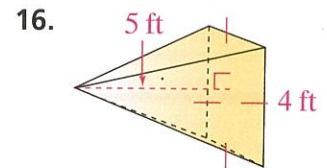
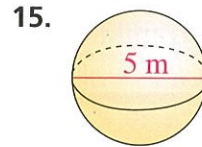
Name the space figure for each net.



Find the surface area of each figure.



Find the volume of each figure to the nearest cubic unit.



19. The height of a rectangle is doubled while the base is unchanged. How does this affect the area? Explain.
20. In cubic feet, how much greater is the volume of a cone with height 10 ft and radius 6 ft than the volume of a square pyramid with height 10 ft and base edge length 6 ft?
21. The diameter of Mars is about 4,000 mi.  
a. Find the approximate surface area.  
b. Find the approximate volume.
22. A box is 25.5 cm by 17 cm by 5 cm.  
a. How much dry dishwashing detergent can it hold?  
b. Without overlap, how much cardboard is needed to make the box?
23. **Writing in Math** How is the formula for volume of a prism like the formula for volume of a pyramid? How are the formulas different?
24. A rectangular piece of sheet metal measures 26 in. by 20 in. A square measuring 2 in. by 2 in. is cut out of each corner, and the sides are folded to form a box. What is the volume of the box?
25. **Open-Ended** Draw a net for a rectangular prism.

# Test Prep

## CUMULATIVE REVIEW CHAPTERS 1-10

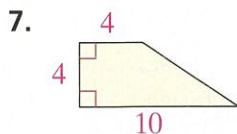
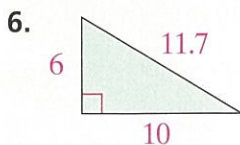
### Multiple Choice

Choose the best answer.

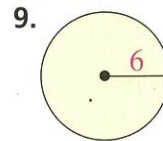
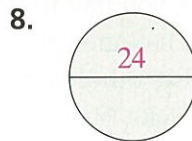
- The average nose has about 6,000,000 cells that detect odors. What is this number in scientific notation?
  - $6 \cdot 10^5$
  - $6 \cdot 10^6$
  - $6 \cdot 10^7$
  - $6 \cdot 10^8$
- Which equation represents the statement *The sum of twice a number and five times another number is 40*?
  - $40 = 2x + 5y$
  - $5y = \frac{40}{2x}$
  - $2x \cdot 5y = 40$
  - $5y = \frac{1}{2}x + 40$
- In which situation are two angles supplementary?
  - The sum of their measures is  $180^\circ$ .
  - They share a vertex.
  - The sum of their measures is  $90^\circ$ .
  - They have the same measure.
- What space figure can you form from the net?
  - square pyramid
  - triangular pyramid
  - triangular prism
  - hexagonal prism
- Figure A is a rectangle 10 in. long and 7.5 in. wide. Figure B is a parallelogram with height 12 in. and base length 7.5 in. Which statement is true?
  - area of A  $>$  area of B
  - area of A  $<$  area of B
  - area of A = area of B
  - cannot be determined

### Gridded Response

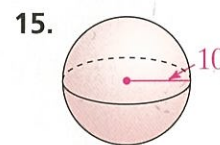
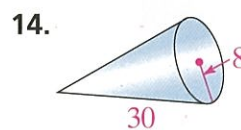
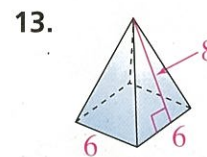
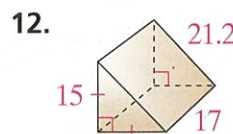
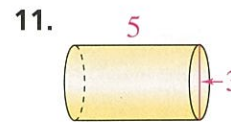
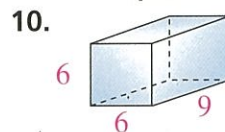
Find each area.



Find each area to the nearest square unit.

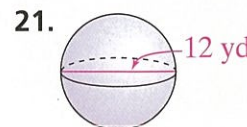
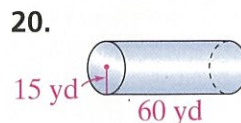
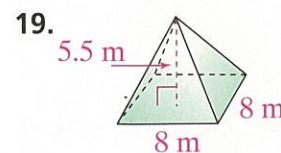
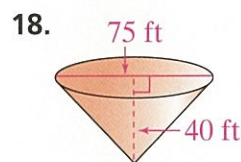
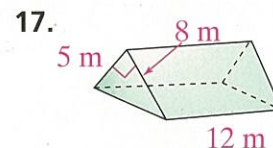
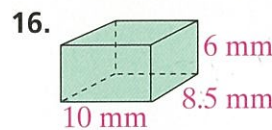


Find the surface area of each figure to the nearest square unit.



### Short Response

Find the volume of each figure, to the nearest cubic unit. Show your work.



### Extended Response

22. A cylinder has diameter 3 cm and height 6 cm. (a) Draw and label a net. (b) Find the surface area of the cylinder to the nearest square centimeter. (c) Find the volume.



### Playing Fields

**Applying Area** Have you ever played a team sport such as soccer, basketball, or ice hockey? If so, you had to follow the rules of the game. You probably found that these include rules about your appearance and the equipment you use, as well as other game equipment such as goals, backboards, or hockey pucks. There are even rules for the space in which you play. One purpose of these rules is to provide all players with an equal chance of success.

#### Football Gridiron

Parallel lines run 5 yd apart from goal line to goal line along the playing area of a football field. The playing area, or gridiron, is 300 ft long.



**Take It to the NET** For more information about sports, go to [www.PHSchool.com](http://www.PHSchool.com).

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